

## Trigonometry and Matrices

### Function of a complex variable

Prove that

$$\log \left( \frac{\sin(x+iy)}{\sin(x-iy)} \right) = 2i \tan^{-1}(\cot x \tanh y)$$

Sol.

$$\begin{aligned} \sin(x+iy) &= \sin x \cos(iy) + \cos x \sin(iy) \\ &= \sin x \cosh y + i \cos x \sinh y \\ &= \alpha + i\beta \end{aligned}$$

$$\log (\sin(x+iy)) = \log (\alpha + i\beta)$$

$$\log (x+iy) = \frac{1}{2} \log (x^2+y^2) + i \tan^{-1} \frac{y}{x}$$

$$\log [\sin(x+iy)] = \frac{1}{2} \log (\alpha^2 + \beta^2) + i \tan^{-1} \frac{\beta}{\alpha} \quad \text{--- (i)}$$

change  $i$  to  $-i$

$$\log [\sin(x-iy)] = \frac{1}{2} \log (\alpha^2 + \beta^2) - i \tan^{-1} \frac{\beta}{\alpha} \quad \text{--- (ii)}$$

$$\text{L.H.S} = \log \left[ \frac{\sin(x+iy)}{\sin(x-iy)} \right]$$

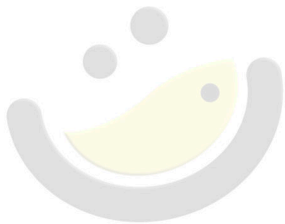
$$= \log [\sin(x+iy)] - \log [\sin(x-iy)]$$

$$= \frac{1}{2} \log(\alpha^2 + \beta^2) + i \tan^{-1} \frac{\beta}{\alpha} - \frac{1}{2} \log(\alpha^2 + \beta^2) + i \tan^{-1} \frac{\beta}{\alpha}$$

$$= 2i \tan^{-1} \frac{\beta}{\alpha}$$

$$= 2i \tan^{-1} \left( \frac{\cos x \sinh y}{\sin x \cosh y} \right)$$

$$= 2i \tan^{-1} (\cot x \tanh y) = \underline{\underline{\text{R.H.S}}}$$



OMG { MATHS }

The poetry of logical ideas.