

Trigonometry and Matrices

Function of a complex variable

$$\sin(\theta + i\phi) = \tan\alpha + i \sec\alpha.$$

show that $\cos 2\theta \cosh 2\phi = 3$

$$\sin(\theta + i\phi) = \tan\alpha + i \sec\alpha$$

Sol.

$$\sin\theta \cos(i\phi) + \cos\theta \sin(i\phi) = \tan\alpha + i \sec\alpha$$

$$\sin\theta \cosh\phi + i \cos\theta \sinh\phi = \tan\alpha + i \sec\alpha$$

Equate real and imaginary parts

$$\tanh d = \sin \theta \cosh d$$

$$\sec d = \cos \theta \sinh d$$

$$\sec^2 d - \tanh^2 d = 1$$

$$\cos^2 \theta \sinh^2 d - \sin^2 \theta \cosh^2 d = 1$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\cosh^2 \theta = \frac{1 + \cosh 2\theta}{2}$$

$$\sinh^2 \theta = \frac{\cosh 2\theta - 1}{2}$$

$$\left(\frac{1 + \cos 2\theta}{2}\right) \left(\frac{\cosh 2\phi - 1}{2}\right) - \left(\frac{1 - \cos 2\theta}{2}\right) \left(\frac{1 + \cosh 2\phi}{2}\right) = 1$$

$$\cosh 2\phi - 1 + \cos 2\theta \cosh 2\phi - \cos 2\theta - \left[1 + \cosh 2\phi - \cos 2\theta - \frac{\cosh 2\phi}{\cos 2\theta}\right] = 4$$

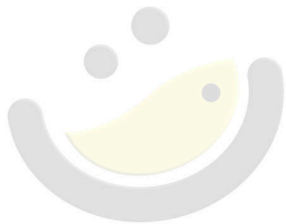
$$\cancel{\cosh 2\phi} - 1 + \cos 2\theta \cosh 2\phi - \cancel{\cos 2\theta} - 1 - \cancel{\cosh 2\phi} + \cancel{\cos 2\theta} + \cosh 2\phi \cos 2\theta = 4$$

$$2 \cos 2\theta \cosh 2\phi - 2 = 4$$

$$2 \cos 2\theta \cosh 2\phi = 6$$

$$\cos 2\theta \cos 2\theta = 3.$$

Hence Proved



OMG { MATHS }
The poetry of logical ideas.