Trigonometry and Matrices

Function of a complex variable

$\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha.$

show that $\cos 2\theta \cosh 2\phi = 3$

Sol:

$\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$

$\sin\theta \cos(i\phi) + \cos\theta \sin(i\phi) = \tan\alpha + i\sec\alpha$

$\sin\theta \cosh\phi + i\cos\theta \sinh\phi = \tan\alpha + i\sec\alpha$

equate real and imaginary parts

$$\tan\alpha = \sin\theta \cosh\phi$$

$$\sec\alpha = \cos\theta \sinh\phi$$

$$\sec^2\alpha - \tan^2\alpha = 1$$

$$\cos^2\theta \sinh^2\phi - \sin^2\theta \cosh^2\phi = 1$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cosh^2\theta = \frac{1+\cosh 2\theta}{2} \qquad \sinh^2\theta = \frac{\cosh 2\theta - 1}{2}$$

$$\left(\frac{1+\cos 2\theta}{2}\right)\left(\frac{\cosh 2\phi-1}{2}\right)-\left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1+\cosh 2\phi}{2}\right)=1$$

$$\cosh 2\phi - 1 + \cos 2\theta \cosh 2\phi - \cos 2\theta - \left[1+\cosh 2\phi - \cos 2\theta - \cosh 2\phi \cos 2\theta\right] = 4$$

$$\cancel{\cosh 2\phi} - 1 + \cos 2\theta \cosh 2\phi - \cancel{\cos 2\theta} - 1 - \cancel{\cosh 2\phi} + \cancel{\cos 2\theta} + \cosh 2\phi \cos 2\theta = 4$$

$$2\cos 2\theta \cosh 2\phi - 2 = 4$$

$$2\cos 2\theta \cosh 2\phi = 6$$

$\cos 2\theta \cosh 2\phi = 3.$

Hence Proved