Trigonometry and Matrices

Function of a complex variable

$$\tan(x+iy) = \cosh(\alpha+i\beta)$$

Prove that

$$\tanh\alpha \tan\beta = \operatorname{cosec} 2x \sinh 2y$$

Sol.

$$\tan(x+iy) = \cosh(\alpha+i\beta)$$

$$\frac{\sin(x+iy)}{\cos(x+iy)} = \cosh(\alpha+i\beta)$$

$$\frac{2\sin(x+iy)\cos(x-iy)}{2\cos(x+iy)\cos(x-iy)} = \cosh\alpha\cosh(i\beta) + \sinh\alpha\sinh(i\beta)$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$\frac{\sin 2x + \sin(2iy)}{\cos 2x + \cos(2iy)} = \cosh\alpha\cos\beta + i\sinh\alpha\sin\beta$$

$$\frac{\sin 2x + i \sinh 2y}{\cos 2x + \cosh 2y} = \cosh\alpha \cos\beta + i \sinh\alpha \sin\beta$$

$$\frac{\sin 2x}{\cos 2x + \cosh 2y} + i \frac{\sinh 2y}{\cos 2x + \cosh 2y} = \cosh\alpha \cos\beta + i \sinh\alpha \sin\beta$$

equate real and imaginary parts

$$\frac{\sin 2x}{\cos 2x + \cosh 2y} = \cosh\alpha \cos\beta \quad \text{—} \quad (1)$$

$$\frac{\sinh 2y}{\cos 2x + \cosh 2y} = \sinh\alpha \sin\beta \quad \text{— (ii)}$$

divide (ii) by (i)

$$\frac{\sinh\alpha \sin\beta}{\cosh\alpha \cos\beta} = \frac{\dfrac{\sinh 2y}{\cancel{\cos 2x + \cosh 2y}}}{\dfrac{\sin 2x}{\cancel{\cos 2x + \cosh 2y}}}$$

$$\frac{\sinh\alpha \sin\beta}{\cosh\alpha \cos\beta} = \frac{\sinh 2y}{\sin 2x}$$

$$\tanh\alpha \tan\beta = \operatorname{cosec} 2x \sinh 2y$$