

Trigonometry and Matrices

Function of a complex variable

$$\tan(x + iy) = \cosh(\alpha + i\beta)$$

Prove that

$$\tanh \alpha \tan \beta = \operatorname{cosec} 2x \sinh 2y$$

$$\tan(x + iy) = \cosh(\alpha + i\beta)$$

Sol.

$$\frac{\sin(x + iy)}{\cos(x + iy)} = \cosh(\alpha + i\beta)$$

$$\frac{2 \sin(x+iy) \cos(x-iy)}{2 \cos(x+iy) \cos(x-iy)} = \cosh a \cosh(iB) + \sinh a \sinh(iB)$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\frac{\sin 2x + \sin(2iy)}{\cos 2x + \cos(2iy)} = \cosh a \cos B + i \sinh a \sin B$$

$$\frac{\sin 2x + i \sinh 2y}{\cos 2x + \cosh 2y} = \cosh \alpha \cos \beta + i \sinh \alpha \sin \beta$$

$$\frac{\sin 2x}{\cos 2x + \cosh 2y} + i \frac{\sinh 2y}{\cos 2x + \cosh 2y} = \cosh \alpha \cos \beta + i \sinh \alpha \sin \beta$$

Equate real and imaginary parts

$$\frac{\sin 2x}{\cos 2x + \cosh 2y} = \cosh \alpha \cos \beta \quad \text{--- (1)}$$

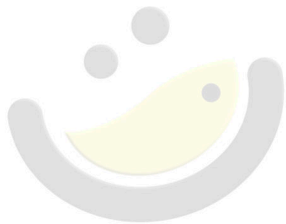
$$\frac{\sinh 2y}{\cosh 2x + \cosh 2y} = \sinh \alpha \sin \beta \quad \text{--- (11)}$$

divide (11) by (1)

$$\frac{\sinh \alpha \sin \beta}{\cosh \alpha \cos \beta} = \frac{\sinh 2y}{\cosh 2x + \cosh 2y} \cdot \frac{\sin 2x}{\cosh 2x + \cosh 2y}$$

$$\frac{\sinh \alpha \sin \beta}{\cosh \alpha \cos \beta} = \frac{\sinh 2\gamma}{\sin 2x}$$

$$\underline{\underline{\tanh \alpha \tan \beta = \operatorname{cosec} 2x \sinh 2\gamma}}$$



OMG! MATHS }
The poetry of logical ideas.