Trigonometry and Matrices

Function of a complex variable

If $x+iy = \tan(A+iB)$

Prove that

$$x^2+y^2-2y\coth 2B+1=0$$

Sol:

$\tan(A+iB) = x+iy. \quad — \text{(I)}$

Change i to $-i$

$\tan(A-iB) = x-iy \quad — \text{(II)}$

$$\tan(2iB) = \tan\{A + iB - A + iB\}$$

$$= \tan\{(A + iB) - (A - iB)\}$$

$$i\tanh(2B) = \frac{\tan(A + iB) - \tan(A - iB)}{1 + \tan(A + iB)\tan(A - iB)}$$

$$i\tanh 2B = \frac{x + iy - (x - iy)}{1 + (x + iy)(x - iy)} \quad [\text{from } (I) \text{ \& } (II)]$$

$$= \frac{\cancel{x} + iy - \cancel{x} + iy}{1 + x^2 - i^2y^2}$$

$$\cancel{i}\tanh(2B) = \frac{2\cancel{i}y}{1+x^2+y^2}$$

$$\tanh 2B = \frac{2y}{1+x^2+y^2}$$

$$\coth 2B = \frac{1+x^2+y^2}{2y}$$

$$2y\coth 2B = 1+x^2+y^2$$

$$x^2+y^2-2y\coth 2B+1=0$$