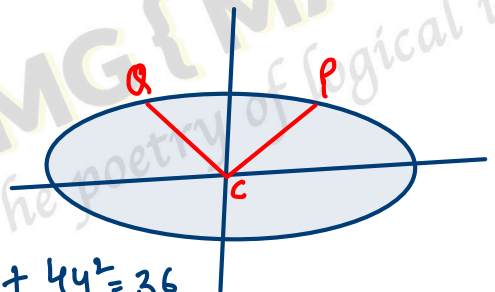
Plane Geometry

Ellipse

Find the length of the semi-diameter conjugate to the diameter y = 3x of the ellipse $9x^2 + 4y^2 = 36$



Sol. given ellipse is

$9x^2 + 4y^2 = 36$

$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a^2 = 4 \quad b^2 = 9.$

Sum of squares of Conjugate semi-diameter is $a^2 + b^2$

$4 + 9 = 13 \quad \text{—} (1)$

Given semi-diameter $y = 3x$ intersect the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

So $\frac{x^2}{4} + \frac{4x^2}{4} = 1$

$\frac{x^2 + 4x^2}{4} = 1$

$5x^2 = 4$

$x^2 = \frac{4}{5}$

$x = \pm\sqrt{\frac{4}{5}}$

$x = \pm\frac{2}{\sqrt{5}}$

$$y = \pm \frac{6}{\sqrt{5}}$$

$\therefore$ P is $\left(\frac{2}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right)$

$$\therefore PC = \sqrt{\left(\frac{2}{\sqrt{5}} - 0\right)^2 + \left(\frac{6}{\sqrt{5}} - 0\right)^2}$$

$$= \sqrt{\frac{4}{5} + \frac{36}{5}} = \sqrt{\frac{40}{5}}$$

$$= \sqrt{8} = 2\sqrt{2}$$

$$PC = 2\sqrt{2}$$

$$(PC)^2 + (QC)^2 = 13 \quad [\text{from } ①]$$

$$(2\sqrt{2})^2 + (QC)^2 = 13$$

$$8 + (QC)^2 = 13$$

$$(QC)^2 = 13 - 8$$

$$(QC)^2 = 5$$

$$QC = \sqrt{5}$$ which is required result.