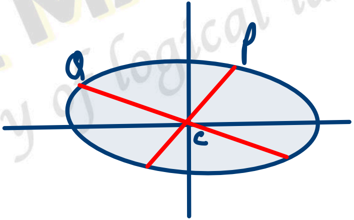
Plane Geometry

Ellipse

Prove that the eccentric angles of the extremities of two conjugate semi Diameters of an ellipse differ by a right angle



Let θ_1, θ_2 be the eccentric angles of P and Q.

$\therefore$ P $(a\cos\theta_1, b\sin\theta_1)$

Q $(a\cos\theta_2, b\sin\theta_2)$

$$\text{slope of } CP = \frac{b\sin\theta_1 - 0}{a\cos\theta_1 - 0}$$

$$= \frac{b}{a}\tan\theta_1$$

$$\text{slope of } CQ = \frac{b\sin\theta_2 - 0}{a\cos\theta_2 - 0}$$

$$= \frac{b}{a}\tan\theta_2$$

Also CP, CD are conjugate diameter

$$m_1 m_2 = -\frac{b^2}{a^2}$$

$$\Rightarrow \left(\frac{b}{a}\tan\theta_1\right)\left(\frac{b}{a}\tan\theta_2\right) = \frac{-b^2}{a^2}$$

$$\Rightarrow \tan\theta_1 \tan\theta_2 = -1$$

$$\Rightarrow \frac{\sin\theta_1}{\cos\theta_1} \cdot \frac{\sin\theta_2}{\cos\theta_2} = -1$$

$$\Rightarrow \sin\theta_1 \sin\theta_2 = -\cos\theta_1 \cos\theta_2$$

$$\Rightarrow \sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2 = 0$$

$$\Rightarrow \cos(\theta_1 - \theta_2) = 0$$

$$\Rightarrow \theta_1 - \theta_2 = \pi/2$$

Hence Proved.