Plane Geometry

Ellipse

Prove that locus of the point whose polar w.r.t the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touches the parabola $y^2=4kx$ is another parabola.

Sol. Given ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Let pole is (x_1, y_1)

eq. of Polar

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad -①$$

Given that ① is tangent to Parobola $y^2 = 4kx$

Condition for tangent to parabola

$$nl = am^2$$

$$(-1)\frac{x_1}{a^2} = k \cdot \frac{y_1^2}{b^4}$$

$$\frac{(-1)x_4}{a^2} \times \frac{b^4}{k} = y_1^2$$

$$y_1^2 = \left(-\frac{b^4}{a^2 k}\right) x_4$$

Which is a parabola.

Hence Proved.