Plane Geometry Ellipse
Find the equation of tangents drawn from the point $(4,1)$ to the ellipse $x^{2}+2 y^{2}=6$ and Prove that the angle between them is $\tan ^{-1} \frac{3}{2}$

sor Given ellipse is

$$
\begin{align*}
& x^{2}+2 y^{2}=6 \\
& \frac{x^{2}}{6}+\frac{2 y^{2}}{6}=1 \\
& \frac{x^{2}}{6}+\frac{y^{2}}{3}=1 \tag{1}
\end{align*}
$$

e). of tangent at (1)

$$
y=m x+\sqrt{a^{2} m^{2}+b^{2}}
$$

$$
\begin{equation*}
y=m x+\sqrt{6 m^{2}+3} \tag{1}
\end{equation*}
$$

el. (11) Passes through $(4,1)$ (given)

$$
\begin{aligned}
& 1=4 m+\sqrt{6 m^{2}+3} \\
& 1-4 m=\sqrt{6 m^{2}+3}
\end{aligned}
$$

sp. both side

$$
\begin{gathered}
(1-4 m)^{2}=6 m^{2}+3 \\
1+16 m^{2}-8 m-6 m^{2}-3=0 \\
10 m^{2}-8 m-2=0
\end{gathered}
$$

$$
\begin{gathered}
5 m^{2}-4 m-1=0 \\
5 m^{2}-5 m+m-1=0 \\
5 m(m-1)+1(m-1)=0 \\
(5 m+1)(m-1)=0 \\
5 m+1=04 m-1=0 \\
m=-1 / 5,1 . \\
m_{1}=\frac{-1}{5} \quad m_{2}=1 .
\end{gathered}
$$

for $m=-1 / s$
e. of tangent is

$$
\begin{aligned}
& y=m x+\sqrt{6 m^{2}+3} \\
& y=\frac{-1}{5} x+\sqrt{6 \times \frac{1}{25}+3} \\
& y+\frac{1}{5} x=\sqrt{\frac{6+75}{25}} \\
& y+\frac{1}{5} x=\sqrt{\frac{81}{25}} \\
& y+\frac{1}{5} x=\frac{9}{5}
\end{aligned}
$$

$$
\begin{aligned}
& y+\frac{1}{5} x-\frac{9}{5}=0 \\
& x+5 y-9=0
\end{aligned}
$$

for $m=1$

$$
\begin{aligned}
& y=x+\sqrt{6+3} \\
& y-x=\sqrt{9} \\
& y-x=3 \\
& y-x-3=0
\end{aligned}
$$

$$
\begin{aligned}
& x-y+3=0 \\
& \tan \theta=\frac{\left|\frac{-1}{5}-1\right|}{1+\left(\frac{-1}{5}\right)(1)} \tan \theta=\left|\frac{m_{1}-m_{2}}{1+m_{1} m_{2}}\right| \\
& =\left|\frac{\frac{-1-5}{5}}{1-\frac{1}{5}}\right|=\left|\frac{\frac{-6}{8}}{\frac{4}{5}}\right|
\end{aligned}
$$

$$
\begin{aligned}
\tan \theta & =\frac{3}{2} \\
\theta & =\tan ^{-1} \frac{3}{2}
\end{aligned}
$$

Hence Proved.

