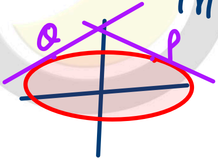
Plane Geometry

Ellipse

If the eccentric angle of two points on an ellipse differ by $\pi/2$ then show that the tangents to the curve at these points intersect on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$



Sol. Let eccentric angle of P is θ.

So eccentric angle of Q is $\pi/2+\theta$

tangent at P to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1 \quad \text{—} (1)$$

tangent at Q to ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$\frac{x}{a}\cos\left(\frac{\pi}{2}+\theta\right)+\frac{y}{b}\sin(\pi/2+\theta)=1 \quad \text{—(II)}$$

squaring and adding (I) & (II)

$$\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2+\left(\frac{x}{a}\cos(\pi/2+\theta)+\frac{y}{b}\sin(\pi/2+\theta)\right)^2=1+1$$

$$\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2+\left(\frac{x}{a}(-\sin\theta)+\frac{y}{b}\cos\theta\right)^2=2$$

$$\frac{x^2}{a^2}\cos^2\theta + \frac{y^2}{b^2}\sin^2\theta + 2\cdot\cancel{\frac{x}{a}\cos\theta}\cdot\frac{y}{b}\sin\theta +$$

$$\frac{x^2}{a^2}\sin^2\theta + \frac{y^2}{b^2}\cos^2\theta - 2\cdot\cancel{\frac{x}{a}\sin\theta}\cdot\frac{y}{b}\cos\theta = 2$$

$$\frac{x^2}{a^2}(\cos^2\theta + \sin^2\theta) + \frac{y^2}{b^2}(\sin^2\theta + \cos^2\theta) = 2$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2.$$

Which is required result.