Plane Geometry

Ellipse

Show that the condition that the pole of $lx + my = 1$ w.r.t the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ may lie on the ellipse $\frac{x^2}{9a^2} + \frac{y^2}{9b^2} = 1$ is

$$a^2l^2 + b^2m^2 = 9$$

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Sol Given ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Let (x_1, y_1) be the pole.

eq. of Polar

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad \text{— (I)}$$

But given polar is

$$lx + my = 1 \quad \text{— (II)}$$

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$\therefore$ eq. (i) & (ii) are same

$$\frac{x_1}{a^2} = l \quad \& \quad \frac{y_1}{b^2} = m.$$

$$x_1 = l a^2 \quad \& \quad y_1 = m b^2 \quad \text{--- (iii)}$$

also (x_1, y_1) lie on

$$\frac{x^2}{9a^2} + \frac{y^2}{9b^2} = 1$$

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$$\frac{x_1^2}{9a^2} + \frac{y_1^2}{9b^2} = 1$$

$$\frac{l^2 a^{\cancel{4}\,2}}{9\cancel{a^2}} + \frac{m^2 b^{\cancel{4}\,2}}{9\cancel{b^2}} = 1 \quad \{\text{from (iii)}\}$$

$$\frac{l^2 a^2}{9} + \frac{m^2 b^2}{9} = 1$$

Hence Proved $\quad l^2 a^2 + m^2 b^2 = 9$

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