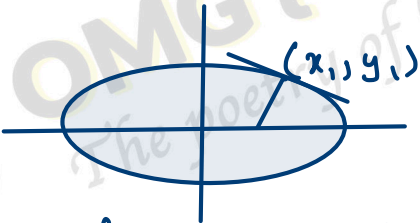
Plane Geometry

Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Equation of Tagent at (x_1, y_1)

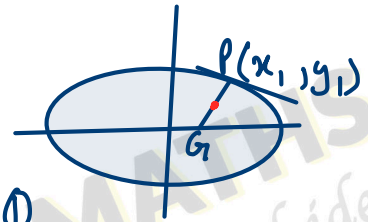
$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

equation of Normal.

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$$

Q1 If the normal at any point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets its major axis in G. find the locus of mid-point of the Chord PG.

Sol Given ellipse is



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad -(1)$$

eq. of Normal to ellipse (1) is

$$\frac{a^2 x}{x_1} - \frac{b^2 y}{y_1} = a^2 - b^2 \quad -(11)$$

(11) meets the major axis $y=0$ at G.

Point G is $\frac{a^2 x}{x_1} = a^2 - b^2$

$$\frac{a^2x}{x_1} = a^2 - a^2(1-e^2)$$

$[\because b^2 = a^2(1-e^2)]$

$$\frac{a^2x}{x_1} = a^2[1 - x + e^2]$$

$$\frac{a^2x}{x_1} = a^2e^2$$

$$x = x_1e^2$$

So Point G $(x_1e^2, 0)$

Mid Point of $P(x_1, y_1)$ & $G(x_1 e^2, 0)$ is

$$X = \frac{x_1 + x_1 e^2}{2} \qquad Y = \frac{y_1 + 0}{2}$$

$$X = \frac{x_1 (1 + e^2)}{2} \qquad Y = \frac{y_1}{2}$$

$$2X = x_1 (1 + e^2) \qquad 2Y = y_1$$

$$\frac{2X}{1 + e^2} = x_1$$

(x_1, y_1) lies on ellipse ①

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$$

$$\frac{\frac{4x^2}{(1+e^2)^2}}{a^2} + \frac{4y^2}{b^2} = 1$$

$$\frac{4x^2}{a^2(1+e^2)} + \frac{4y^2}{b^2} = 1$$

Which is Required locus of mid point of PQ.