

Plane Geometry

Circle

Parametric representation of Circle

Exp show that the point (x, y) given by

$$x = \frac{20t}{4+t^2}, y = \frac{5(4-t^2)}{4+t^2}$$

lies on circle for all real values of t
(or)

Eliminate the parameter t from the equations

$$x = \frac{20t}{4+t^2}, y = \frac{5(4-t^2)}{4+t^2}$$

~~$x =$~~

$$x^2 + y^2 = \left(\frac{20t}{4+t^2} \right)^2 + \left(\frac{5(4-t^2)}{4+t^2} \right)^2$$

$$= \frac{400t^2 + 25(4-t^2)^2}{(4+t^2)^2}$$

~~$=$~~

$$\frac{400t^2 + 25[16 + t^4 - 8t^2]}{(4+t^2)^2}$$

$$= \frac{400t^2 + 400 + 25t^4 - 200t^2}{(4+t^2)^2}$$

$$= \frac{25t^4 + 200t^2 + 400}{(4+t^2)^2}$$

$$= \frac{25(t^4 + 8t^2 + 16)}{(4+t^2)^2}$$

$$= \frac{25}{(t^2 + 4)^2}$$

$$x^2 + y^2 =$$

$$\frac{25}{(t^2 + 4)^2}$$

$$\underline{\underline{x^2 + y^2 = a^2}}$$

$$x^2 + y^2 = 5^2$$