Plane Geometry

Circle

Parametric representation of Circle

(i) Express the Co-ordinates of any point on the Circle $x^2 + y^2 = a^2$ in terms of One Variable (Called the parameter)

(ii) Express the Co-ordinates of any point on Circle $(x-h)^2 + (y-k)^2 = a^2$ in terms of One Variable.
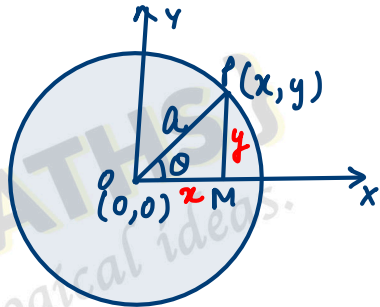
(i) Sol.

$$x^2 + y^2 = a^2$$

In $\triangle OPM$

$$\cos\theta = \frac{OM}{OP}$$

$$\cos\theta = \frac{x}{a}$$

$$x = a\cos\theta \quad —①$$

$$\sin\theta = \frac{PM}{OP} = \frac{y}{a}$$

$$y = a \sin\theta \quad —②$$

from ① & ②

$$x = a \cos\theta \ \& \ y = a \sin\theta. \ [0 \leqslant \theta \leqslant 2\pi]$$

(ii) Sol.

eq. is Circle is

$$(x-h)^2 + (y-k)^2 = a^2$$

let $x-h = x' \quad y-k = y'$

$$x'^2 + y'^2 = a^2$$

$$x' = a\cos\theta \qquad y' = a\sin\theta$$

$$(0 \leq \theta = 2\pi)$$

$$x - h = a\cos\theta \quad , \quad y - k = a\sin\theta$$

$$x = h + a\cos\theta$$
$$y = k + a\sin\theta.$$

Find the parametric eq. of the circle

$$x^2 + y^2 - 2x + 4y - 4 = 0$$

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) - 4 - 1 - 4 = 0$$

$$(x-1)^2 + (y+2)^2 = 9$$

$$(x-1)^2 + (y+2)^2 = (3)^2$$
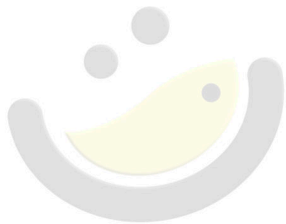
Compare with $(x-h)^2 + (y-k)^2 = a^2$

$$h = 1 \quad , \quad K = -2 \quad , \quad a = 3.$$

Parametric eq. of Circle.

$$x = 1 + 3\cos\theta$$
$$y = -2 + 3\sin\theta \qquad \left[0 \leqslant \theta \leqslant 2\pi \right]$$