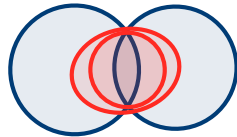# Plane Geometry

Coaxial system of Circles

Find the limiting points of the Co-axial system determined by the Circles.

$$x^2 + y^2 - 6x - 6y + 4 = 0$$

$$x^2 + y^2 - 2x - 4y + 3 = 0$$

sol. Given Circles are

$$x^2 + y^2 - 6x - 6y + 4 = 0 \quad —\text{①}$$

$$x^2 + y^2 - 2x - 4y + 3 = 0 \quad —\text{⑪}$$

subtract ⑪ from ①

$$\cancel{x^2} + \cancel{y^2} - 6x - 6y + 4 - \cancel{x^2} - \cancel{y^2} + 2x + 4y - 3 = 0$$

$$-4x - 2y + 1 = 0$$

$$4x + 2y - 1 = 0$$

Now eq. of point circle co-axial with①

$$(x^2 + y^2 - 6x - 6y + 4) + \lambda(4x + 2y - 1) = 0$$

$$x^2 + y^2 - 6x - 6y + 4 + 4\lambda x + 2\lambda y - \lambda = 0$$

$$x^2 + y^2 + (-6 + 4\lambda)x + (-6 + 2\lambda)y + (4 - \lambda) = 0$$

Compare with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$g = \frac{-6 + 4\lambda}{2}, \quad f = \frac{-6 + 2\lambda}{2}, \quad c = 4 - \lambda$$

$$g = -3 + 2\lambda \quad, \quad f = \left(\dfrac{-3+\lambda}{}\right), \quad c = 4-\lambda$$

Centre $(-g, -f)$

$$= (3 - 2\lambda, \; 3 - \lambda)$$

Radius :- $\sqrt{g^2 + f^2 - c}$

$$= \sqrt{(-3 + 2\lambda)^2 + (-3 + \lambda)^2 - (4-\lambda)}$$

for limiting point   Radius $= 0$

$$\sqrt{(-3+2\lambda)^2 + (-3+\lambda)^2} - (4-\lambda) = 0$$

$$(-3+2\lambda)^2 + (-3+\lambda)^2 - (4-\lambda) = 0$$

$$9 + 4\lambda^2 - 12\lambda + \lambda^2 + 9 - 6\lambda - 4 + \lambda = 0$$

$$5\lambda^2 - 17\lambda + 14 = 0$$

$$5\lambda^2 - 10\lambda - 7\lambda + 14 = 0$$

$$5\lambda(\lambda - 2) - 7(\lambda - 2) = 0$$

$$(5\lambda - 7)(\lambda - 2) = 0$$

$$\lambda = 7/5, \; 2.$$

for $\lambda = \dfrac{7}{5}$

centre. $\left(3 - 2\lambda, \; 3 - \lambda\right)$

$$\left(\left(3 - \dfrac{14}{5}\right), \left(3 - \dfrac{7}{5}\right)\right)$$

$$\left(\dfrac{15 - 14}{5}, \; \dfrac{15 - 7}{5}\right) = \left(\dfrac{1}{5}, \dfrac{8}{5}\right)$$
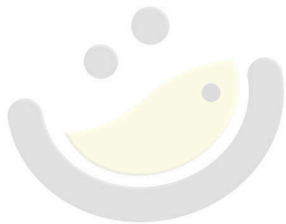
When $\lambda = 2$

Centre $= (3-4, 3-2)$

$(-1, 1)$

limiting Points are

$\left(\dfrac{1}{5}, \dfrac{8}{5}\right) ; (-1, 1)$

Find the Radical axis and the limiting point of the system of Co-axial Circles.

$$\left(3(x^2 + y^2) - 16x - 14y + 39\right) + \lambda$$
$$\left(x^2 + y^2 - 5x - 5y + 13\right) = 0$$