

Plane Geometry

Coaxial system of Circles

Find the Radical axis and limiting points of the Co-axial system determined by

$$x^2 + y^2 + 2x - 6y = 0$$

$$2x^2 + 2y^2 - 10y + 5 = 0$$

$$x^2 + y^2 + 2x - 6y = 0 \quad \text{--- ①}$$

$$2x^2 + 2y^2 - 10y + 5 = 0$$

divide by 2

Proof

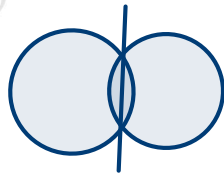
$$x^2 + y^2 - 5y + 5/2 = 0 \quad \text{--- (2)}$$

Subtract (2) from (1)

$$\cancel{x^2} + \cancel{y^2} + 2x - 6y - \cancel{x^2} - \cancel{y^2} + 5y - 5/2 = 0$$

$$2x - y - 5/2 = 0$$

$$4x - 2y - 5 = 0$$



which is Radical axis.

The equation of Circle Co-axial with Circle (1)

$$(x^2 + y^2 + 2x - 6y) + \lambda(4x - 2y - 5) = 0$$

$$x^2 + y^2 + 2x - 6y + 4\lambda x - 2\lambda y - 5\lambda = 0$$

$$x^2 + y^2 + (2 + 4\lambda)x + (-6 - 2\lambda)y - 5\lambda = 0$$

Compare it with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$g = \frac{2 + 4\lambda}{2} = 2\lambda + 1$$

$$f = \frac{-6-2\lambda}{2} = -(3+\lambda)$$

$$c = -5\lambda.$$

Centre of Circle $(-g, -f)$

$$(-(2\lambda+1), 3+\lambda)$$

Radius of Circle.

$$= \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{(2\lambda+1)^2 + (-(3+\lambda))^2 + 5\lambda}$$

for limiting point Radius = 0

$$\sqrt{(2\lambda+1)^2 + (-(3+\lambda))^2} + 5\lambda = 0$$

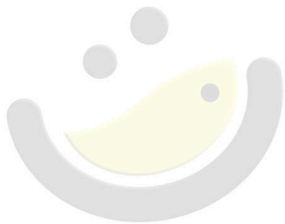
$$4\lambda^2 + 1 + 4\lambda + 9 + \lambda^2 + 6\lambda + 5\lambda = 0$$

$$5\lambda^2 + 15\lambda + 10 = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$\lambda^2 + 2\lambda + \lambda + 2 = 0$$

$$\lambda(\lambda+2) + 1(\lambda+2) = 0$$



$$(\lambda + 1)(\lambda + 2) = 0$$

$$\lambda = -1, -2$$

Centre . $(-(2\lambda + 1), 3 + \lambda)$

for $\lambda = -1$

Centre = $(1, 2)$

for $\lambda = -2$

Centre = $(3, 1)$

limiting points

$(1, 2)$ $(3, 1)$