

Plane Geometry

Coaxial system of Circles

If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is a circle of a Co-axial system having the origin as its limiting point, show that other limiting point is

$$\left(\frac{-gc}{g^2 + f^2}, \frac{-fc}{g^2 + f^2} \right)$$



Proof

limiting point is Centre of point
Circle of Co-axial system.

$(0,0)$ is limiting point (given)

eq. of point Circle

$$(x-0)^2 + (y-0)^2 = 0$$

$$x^2 + y^2 = 0 \quad \text{--- ①}$$

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{--- ②}$$

subtract ① from ②

$$\cancel{x^2} + \cancel{y^2} + 2gx + 2fy + c - \cancel{x^2} - \cancel{y^2} = 0$$

$$2gx + 2fy + c = 0$$

which is Radical axis of \odot & \odot'

eq. of Circle Co axial with \odot is

$$(x^2 + y^2) + \lambda(2gx + 2fy + c) = 0$$

$$x^2 + y^2 + 2g\lambda x + 2f\lambda y + \lambda c = 0$$

$$\text{Centre} \therefore (-g\lambda, -f\lambda)$$

$$\text{Radius} = \sqrt{g^2 \lambda^2 + f^2 \lambda^2 - c \lambda}$$

for limiting point

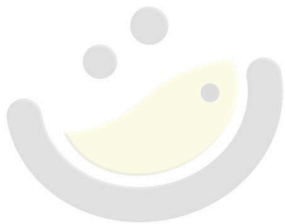
$$\text{Radius} = 0$$

$$\sqrt{g^2 \lambda^2 + f^2 \lambda^2 - c \lambda} = 0$$

$$g^2 \lambda^2 + f^2 \lambda^2 - c \lambda = 0$$

$$\lambda (g^2 \lambda + f^2 \lambda - c) = 0$$

$$\lambda = 0 \quad (g^2 + f^2) \lambda = c$$



$$\lambda = \frac{c}{g^2 + f^2}$$

When $\lambda = 0$

Centre $(0, 0)$

When $\lambda = \frac{c}{g^2 + f^2}$

Centre = $\left(\frac{-gc}{g^2 + f^2}, \frac{-fc}{g^2 + f^2} \right)$

Other

limiting

points is

$\left(\frac{-gc}{g^2 + f^2}, \frac{-fc}{g^2 + f^2} \right)$