Plane Geometry

Coaxial system of Circles

If the circle $x^2+y^2+2gx+2fy+c=0$ is a circle of a co-axial system having the origin as its limiting point, show that other limiting point is

$$\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2}\right)$$

Proof

limiting point is Centre of point Circle of Co-axial system.

$(0,0)$ is limiting point (given)

eq. of point Circle

$$(x-0)^2 + (y-0)^2 = 0$$

$$x^2 + y^2 = 0 \quad \text{--- (I)}$$

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \text{--- (II)}$$

Subtract (I) from (II)

$$\cancel{x^2}+\cancel{y^2}+2gx+2fy+c-\cancel{x^2}-\cancel{y^2}=0$$

$$2gx+2fy+c=0$$

Which is Radical axis of (I) & (II)

eq. of circle co axial with (I) is

$$(x^2+y^2)+\lambda(2gx+2fy+c)=0$$

$$x^2+y^2+2g\lambda x+2f\lambda y+\lambda c=0$$

Centre : $(-g\lambda, -f\lambda)$

Radius $= \sqrt{g^2\lambda^2 + f^2\lambda^2 - c\lambda}$

for limiting point

Radius $= 0$

$$\sqrt{g^2\lambda^2 + f^2\lambda^2 - c\lambda} = 0$$

$$g^2\lambda^2 + f^2\lambda^2 - c\lambda = 0$$

$$\lambda\,(g^2\lambda + f^2\lambda - c) = 0$$

$$\lambda = 0 \qquad (g^2 + f^2)\lambda = c$$

$$\lambda = \frac{c}{g^2+f^2}$$

When $\lambda = 0$

Centre $(0,0)$

When $\lambda = \frac{c}{g^2+f^2}$

Centre $= \left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2} \right)$

Other limiting points is $\left(\frac{-gc}{g^2+f^2}, \frac{-fc}{g^2+f^2} \right)$