Plane Geometry

Pair of straight lines

Find the angle between the pair of straight lines represented by

$$x^2 + xy - 6y^2 + 7x + 31y - 18 = 0$$

Also find the equation of the pair of straight lines parallel to these and passing through the point $(1, 2)$

Sol. Given eq. is

$$x^2 + xy - 6y^2 + 7x + 31y - 18 = 0 \quad \text{—①}$$

Compare ① with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

$a = 1$, $h = 1/2$, $b = -6$, $g = 7/2$, $f = 31/2$

$c = -18$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}$$

$$= \frac{2\sqrt{\frac{1}{4} - (1)(-6)}}{|1 + (-6)|}$$

$$= \frac{2\sqrt{\frac{1}{4} + 6}}{5}$$

$$= \frac{2\sqrt{\frac{25}{4}}}{5} = \frac{2 \times \frac{5}{2}}{5} = \frac{5}{5} = 1$$

$\tan\theta = 1.$

$\theta = 45^\circ$

$x^2 + xy - 6y^2 + 7x + 31y - 18 = 0$

$x^2 + (y+7)x + (-6y^2 + 31y - 18) = 0$

$$x = \frac{-(y+7) \pm \sqrt{(y+7)^2 - 4(-6y^2+31y-18)}}{2}$$

$$x = \frac{-y-7 \pm \sqrt{y^2+49+14y+24y^2-124y+72}}{2}$$

$$x = \frac{-y-7 \pm \sqrt{25y^2-110y+121}}{2}$$

$$= \frac{-y-7 \pm \sqrt{(5y-11)^2}}{2}$$

$$= \frac{-y - 7 \pm (5y - 11)}{2}$$

$$x = \frac{-y - 7 + 5y - 11}{2} \quad \text{and}$$

$$x = \frac{-y - 7 - 5y + 11}{2}$$

$$\Rightarrow x = \frac{+4y - 18}{2} \quad \text{and} \quad \frac{-6y + 4}{2}$$

$$x = 2y - 9 \quad \text{and} \quad -3y + 2$$

$$x - 2y + 9 = 0 \quad - (II)$$

$$x + 3y - 2 = 0 \quad - (III)$$

The Line Parallel to (III) is

$$x - 2y + k = 0 \quad - (IV)$$

(IV) Passes through (1, 2) (Given)

$$1 - 2(2) + k = 0$$

$$1 - 4 + k = 0$$

$$k = 3.$$

$x - 2y + 3 = 0$

eq. parallel to (iii) is

$x + 3y + \lambda = 0$ — (v)

(v) passes through $(1, 2)$ (given)

$1 + 3(2) + \lambda = 0$

$1 + 6 + \lambda = 0$

$7 + \lambda = 0 \Rightarrow \lambda = -7$

$$x + 3y - 7 = 0$$

$$x - 2y + 3 = 0$$

$$(x + 3y - 7)(x - 2y + 3) = 0$$

$$x^2 - 2xy + 3x + 3xy - 6y^2 + 9y - 7x + 14y - 21 = 0$$

$$\boxed{x^2 + xy - 4x - 6y^2 + 23y - 21 = 0}$$