Plane Geometry

Pair of straight lines

If p_1 and p_2 are the lengths of perpendiculars drawn from the point $(-1,2)$ to the pair of lines given by the equation.

$$2x^2 - 5xy + 2y^2 + 3x - 3y + 1 = 0$$

Prove that $p_1 p_2 = \frac{12}{5}$

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Sol. Given eq. is

$$2x^2 - 5xy + 2y^2 + 3x - 3y + 1 = 0 \quad -(1)$$

$$2x^2 + (-5y + 3)x + (2y^2 - 3y + 1) = 0$$

This is Quad. in x.

$$x = \frac{-(-5y+3) \pm \sqrt{(-5y+3)^2 - 4(2)(2y^2-3y+1)}}{2(2)}$$

$$x = \frac{5y - 3 \pm \sqrt{25y^2 + 9 - 30y - 16y^2 + 24y - 8}}{4}$$

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$$x = \frac{5y - 3 \pm \sqrt{9y^2 - 6y + 1}}{4}$$

$$x = \frac{5y - 3 \pm \sqrt{(3y-1)^2}}{4}$$

$$x = \frac{5y - 3 \pm (3y-1)}{4}$$

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$$\therefore \quad x = \frac{5y-3+3y-1}{4} \quad \text{and} \quad x = \frac{5y-3-(3y-1)}{4}$$

$$\Rightarrow \quad x = \frac{8y-4}{4} \quad \text{and} \quad x = \frac{2y-2}{4}$$

$$\Rightarrow \quad x = 2y-1 \quad \text{and} \quad x = \frac{y-1}{2}$$

$$\Rightarrow \quad x - 2y + 1 = 0 \quad (\text{I}) \quad \text{and} \quad 2x - y + 1 = 0 \quad (\text{II})$$

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eq. (i) and (ii) are straight lines given by (i)

p_1 = distance of $(-1,2)$ from (i)

$$p_1 = \frac{|(-1) - 2(2) + 1|}{\sqrt{1+4}} = \frac{4}{\sqrt{5}}$$

p_2 = distance of $(-1,2)$ from (ii)

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$$p_2 = \frac{|2(-1) - (2) + 1|}{\sqrt{4+1}}$$

$$= \frac{3}{\sqrt{5}}$$

$$p_1 p_2 = \frac{4}{\sqrt{5}} \times \frac{3}{\sqrt{5}} = \frac{12}{5}$$

Hence proved.

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