Plane Geometry

Pair of straight lines

Show that the equations

$$x^2 + 2\sqrt{3}\,xy + 3y^2 - 3x - 3\sqrt{3}\,y - 4 = 0$$

represent a pair of parallel straight lines. Find the distance between them.

Sol. Given eq. is

$$x^2 + 2\sqrt{3}\,xy + 3y^2 - 3x - 3\sqrt{3}\,y - 4 = 0$$

$$x^2 + (2\sqrt{3}y - 3)x + (3y^2 - 3\sqrt{3}y - 4) = 0$$

$$x = \frac{-(2\sqrt{3}y - 3) \pm \sqrt{(2\sqrt{3}y - 3)^2 - 4\cdot(3y^2 - 3\sqrt{3}y - 4)}}{2}$$

$$x = \frac{-2\sqrt{3}y + 3 \pm \sqrt{\cancel{12y^2} + 9 - \cancel{12\sqrt{3}y} - \cancel{12y^2} + \cancel{12\sqrt{3}y} + 16}}{2}$$

$$x = \frac{-2\sqrt{3}y + 3 \pm \sqrt{9+16}}{2}$$

$$x = \frac{-2\sqrt{3}y + 3 \pm 5}{2}$$

$$x = \frac{-2\sqrt{3}y + 8}{2} \quad \text{or} \quad x = \frac{-2\sqrt{3}y - 2}{2}$$

either $x = -\sqrt{3}y + 4$ or $x = -\sqrt{3}y - 1$

$\Rightarrow \quad x + \sqrt{3}y - 4 = 0$ — (ii)

$x + \sqrt{3}y + 1 = 0$ — (iii)

Slope of (ii) $= \dfrac{-1}{\sqrt{3}}$

slop of (iii) $= \dfrac{-1}{\sqrt{3}}$

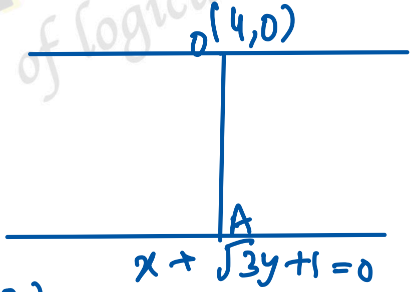
Slope of (ii) = Slope of (iii)

$\therefore$ lines (ii) and (iii) are Parallel

Put $y = 0$ in (2)

$x = 4.$

Point on line (ii) is $(4, 0)$



$$OA = \frac{4+1}{\sqrt{1+3}} = \frac{5}{2} \text{ Ans.}$$