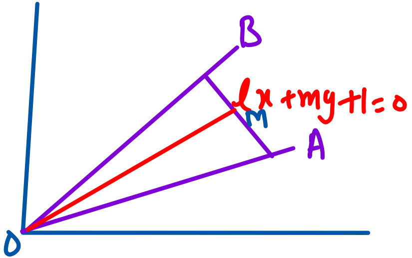
Plane Geometry

Pair of straight lines

Prove that the triangle formed by lines

$ax^2 + 2hxy + by^2 = 0$ and $lx + my + 1 = 0$

is isosceles if $h(l^2 - m^2) = (a-b)lm$



Sol. Given eq. is

$$ax^2 + 2hxy + by^2 = 0 \quad \text{—(I)}$$

And

$$lx + my + 1 = 0 \quad \text{—(II)}$$

The eq. is bisectors of (I) is

$$\frac{x^2-y^2}{a-b} = \frac{xy}{h} \quad \text{--- (III)}$$

The line AB is $lx+my+1=0$

$$m_1 = -\frac{l}{m}$$

Let $OM \perp AB$.

Slope of $OM = \frac{m}{l}$

$\therefore$ eq. of OM is $y = \frac{m}{l}x$ — (4)

ΔAOB is isosceles if one of the bisector given by (11) is $\perp$ to AB.

$\Rightarrow$ if line given by 4 is one of the lines of (3)

$\Rightarrow$ if (4) satisfies (3)

$$\frac{x^2 - \frac{m^2}{l^2}x^2}{a-b} = \frac{x\left(\frac{m}{l}\right)x}{h}$$

$$\Rightarrow \frac{l^2x^2 - m^2x^2}{l^2(a-b)} = \frac{mx^2}{lh}$$

$$\Rightarrow \frac{\cancel{x^2}(l^2 - m^2)}{l^2(a-b)} = \frac{m\cancel{x^2}}{\cancel{l}h}$$

$$\Rightarrow h(l^2 - m^2) = (a-b)lm.$$

Hence <u>Proved.</u>