Plane Geometry

Pair of straight lines

Bisectors of angles between Pair of lines through the Origin

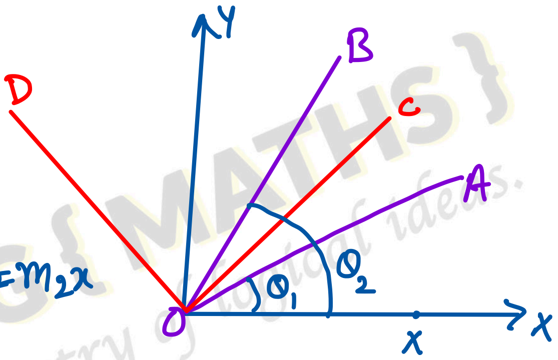
Find the equation to the straight lines bisecting the angles between the straight lines given by

$$ax^2 + 2hxy + by^2 = 0$$

Let $ax^2+2hxy+by^2=0$

represents

$y=m_1x$ and $y=m_2x$



$m_1 = \tan\theta_1$ $m_2 = \tan\theta_2$

$m_1+m_2 = \tan\theta_1+\tan\theta_2 = -\frac{2h}{b}$ — (I)

$m_1m_2 = \tan\theta_1 \cdot \tan\theta_2 = \frac{a}{b}$ — (II)

OC is internal and OD is external bisector of $\angle AOB$.

$$\therefore \quad \angle AOC = \angle COB.$$

$$\angle XOC - \theta_1 = \theta_2 - \angle XOC$$

$$\angle XOC + \angle XOC = \theta_1 + \theta_2$$

$$2\angle XOC = \theta_1 + \theta_2$$

$$\angle XOC = \frac{\theta_1 + \theta_2}{2} \quad \text{--- (iii)}$$

$$\angle XOD = \angle XOC + \angle COD$$

$$= \frac{\theta_1+\theta_2}{2} + \frac{\pi}{2}$$

[internal and external bisectors of an angle are always $\perp$]

$$\angle XOD = \frac{1}{2}\left(\pi + (\theta_1+\theta_2)\right)$$

$$2\angle XOD = \left(\pi + (\theta_1+\theta_2)\right)$$

Let θ is an angle of internal or external bisector with x-axis either $\theta = \angle XOC$ or $\theta = \angle XOD$

$$\theta = \frac{\theta_1 + \theta_2}{2} \quad \text{or} \quad \theta = \frac{1}{2}\{\pi + (\theta_1 + \theta_2)\}$$

$$\theta = \frac{\theta_1 + \theta_2}{2}$$

$$2\theta = \theta_1 + \theta_2$$

$$\tan 2\theta = \tan(\theta_1 + \theta_2)$$

$$= \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2}$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{-\frac{2h}{b}}{1-\frac{a}{b}}$$ [from (I) & (II)]

$$\frac{\cancel{2}\, y/x}{1 - y^2/x^2} = \frac{-\cancel{2}h}{\cancel{b}} \times \frac{\cancel{b}}{b-a}$$

$$y = mx$$

$$m = \frac{y}{x}$$

$$\tan\theta = y/x$$

$$\frac{y}{\cancel{x}} \times \frac{x^{\cancel{2}}}{x^2-y^2} = \frac{h}{a-b}$$

$$\frac{xy}{x^2-y^2} = \frac{h}{a-b}$$

$$\frac{x^2-y^2}{a-b} = \frac{xy}{h}.$$