Pair of straight lines

Show that the equation $bx^2 - 2hxy + ay^2 = 0$ represents a pair of straight lines which are at right angles to the pair given by $ax^2 + 2hxy + by^2 = 0$

Sol. Given eq. is $ax^2 + 2hxy + by^2 = 0$ — (1)

Let eq (1) represents $y = m_1 x$ — (11) and $y = m_2 x$ — (11)

$$m_1 + m_2 = \frac{-2h}{b} \quad - (IV)$$

$$m_1 m_2 = \frac{a}{b} \quad - (V)$$

Now eq. is lines $\perp$ to (II) and (III) are

$$y = \frac{-1}{m_1} x$$

$$y = \frac{-1}{m_2} x$$

$$m_1 y + x = 0$$

$$m_2 y + x = 0$$

Joint eq. is

$$(m_1 y + x)(m_2 y + x) = 0$$

$$m_1 m_2 y^2 + m_1 xy + m_2 xy + x^2 = 0$$

$$m_1 m_2 y^2 + (m_1 + m_2) xy + x^2 = 0$$

$$\left(\frac{a}{b}\right)y^2 + \left(\frac{-2h}{b}\right)xy + x^2 = 0 \quad \left[\text{from } (iv) \text{ and } (v)\right]$$

$$ay^2 - 2hxy + bx^2 = 0$$

Which is the required eq.

Hence Proved.