Plane Geometry

Pair of straight lines

Show that the equation

$$2(x+2)^2 + 3(x+2)(y-2) - 2(y-2)^2 = 0$$

represents two perpendicular lines.

where do they intersect ?

Sol. The given eq.

$$2(x+2)^2 + 3(x+2)(y-2) - 2(y-2)^2 = 0 \quad \text{— (1)}$$

eq. ① is quad in $x+2$

$$x+2 = \frac{-3(y-2) \pm \sqrt{9(y-2)^2 - 4(2)(-2)(y-2)^2}}{2\times 2}$$

$$x+2 = \frac{-3(y-2) \pm \sqrt{(y-2)^2\, 25}}{4}$$

$$4x+8 = -3(y-2) \pm 5(y-2)$$

$$4x + 8 = (y-2)(-3 \pm 5)$$

$$4x + 8 = (y-2)2$$

and $4x + 8 = (y-2)(-8)$

$\Rightarrow$ $4x + 8 - 2y + 4 = 0$

and $4x + 8 + 8y - 16 = 0$

$\Rightarrow$ $4x - 2y + 12 = 0$

$4x + 8y - 8 = 0$

$\Rightarrow \quad 2x - y + 6 = 0 \quad -(II)$

and $\quad x + 2y - 2 = 0 \quad -(III)$

$m_1 =$ slope of $(II) = \frac{-2}{-1} = 2.$

$m_2 =$ slope of $(III) = \frac{-1}{2}$

$m_1 m_2 = 2 \times \frac{-1}{2} = -1$

$m_1 m_2 = -1$

Hence Lines (I) & (II) are $\perp$

$\therefore$ eq. (I) represents two $\perp$ lines.

$$\left.\begin{matrix} 2x - y + 6 = 0 \\ x + 2y - 2 = 0 \end{matrix}\right] \begin{matrix} \times 1 \\ \times 2 \end{matrix}$$

$$2x - y + 6 = 0$$
$$-2x \pm 4y \mp 4 = 0$$

$$-5y + 10 = 0$$

$$-5y = -10$$

$$y = 2$$

$$2x - 2 + 6 = 0$$

$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2$$

point of intersection

$(-2, 2)$