Curve Tracing

Trace the Curve

$$y = x^3 + 5x^2 + 3x - 4.$$

1. Symmetry:- Curve is not symmetrical about x- axis and y- axis.

2. Origin:- Curve does not pass through origin.

3. Domain:- $(-\infty, \infty)$

4. Point of intersection with axis.

for y- axis. Put $x=0$

$y = -4$.

$\therefore$ Curve meets y- axis at $(0,-4)$

for x - axis Put $y=0$

$0 = x^3 + 5x^2 + 3x - 4 \quad — (1)$

$x = -4$ satisfies (1)

By synthetic division

$$\begin{array}{r|rrrr} -4 & 1 & 5 & 3 & -4 \\ & & -4 & -4 & 4 \\ \hline & 1 & 1 & -1 & 0 \end{array}$$

$$x^2 + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 + 2.2}{2}, \frac{-1 - 2.2}{2}$$

$$= 0.6, -1.6$$

$\sqrt{5}$

$\sqrt{4+1}$

$2 + \frac{1}{2 \times 2}$

$2 + \frac{1}{4}$

$2 + 0.25$

2.2

Curve meet x-axis at:

$(-4, 0); (0.6, 0); (-1.6, 0)$

5. <u>Asymptotes</u>: The curve has no asymptote.

6. Increasing and decreasing.

$$y = x^3 + 5x^2 + 3x - 4$$

$$\frac{dy}{dx} = 3x^2 + 10x + 3$$

$$\frac{d^2y}{dx^2} = 6x + 10$$

$$\frac{dy}{dx} > 0$$

$$3x^2 + 10x + 3 > 0$$

$$3x^2 + 9x + x + 3 > 0$$

$$3x(x+3) + 1(x+3) > 0$$

$$(3x+1)(x+3) > 0$$

if x does not lies b/w -3 and $-1/3$.

$\therefore$ Curve is increasing in $(-\infty, -3) \cup (-1/3, \infty)$

Ily Curve is decreasing when $\frac{d^2y}{dx^2} < 0$

Curve is decreasing in $\left(-3, -\frac{1}{3}\right)$

7. Point of inflexion:-

$$\frac{d^2y}{dx^2} > 0$$

$$6x + 10 > 0$$

$$6x > -10$$

$$x > -\frac{10}{6}$$

$$x > -5/3.$$

Concave upward.

$$\frac{d^2y}{dx^2} < 0$$

$$6x + 10 < 0$$

$$6x < -10$$

$$x < -5/3.$$

Concave downward.

$\therefore$ Point of inflexion lies at $x = -5/3$.

$$y = x^3 + 5x^2 + 3x - 4.$$

$$y = \frac{-125}{27} + \frac{5 \times 25}{9} - \frac{15}{3} - 4 = \frac{7}{27}$$

$\left(-5/3, \frac{7}{27}\right)$ is point of inflexion

8. Maxima and minima.

$$\frac{dy}{dx} = 0$$

$$(3x+1)(x+3) = 0$$

$$x = -3, -1/3.$$

for $x = -3$

$$\frac{d^2y}{dx^2} = 6x+10 = 6(-3)+10 = -8 < 0$$

Function has maximum value at $x = -3$.

$y = x^3 + 5x^2 + 3x - 4$

$= -27 + 45 - 9 - 4 = 5$

for $x = -1/3$. $\dfrac{d^2y}{dx^2} = 6x + 10$

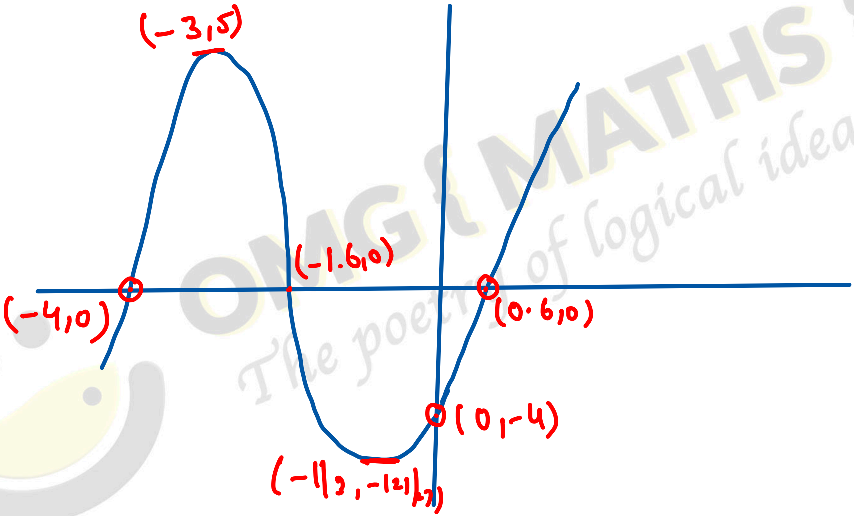
$= 6 \times \dfrac{-1}{3} + 10 = 8 > 0$

$\therefore$ Curve has minima at $x = -1/3$.

$y = -1/27 + 5(1/9) + 3(-1/3) - 4 = \dfrac{-121}{27}$

Additional Points:

1	5
2	30
-1	-3
-2	2
-3	5


(-3,5)
(-1.6,0)
(-4,0)
(0.6,0)
(0,-4)
(-11/3, -121/27)