Curve Tracing

Trace the Curve

$$y = \frac{x}{1+x^2}$$

1. Symmetry:- Curve is not symmetrical about x-axis and y-axis.

2. Origin:- Curve passes through (0,0) origin.

3. Intersection of Curve with axis

Curve intersect at only one point
(0,0)

4. <u>Domain</u>:- $(-\infty, \infty)$

5. <u>Asymptote</u>:- $y=0$ parallel to x-axis

6. <u>Increasing and decreasing</u>:-

$$y = \frac{x}{1+x^2} \quad -(1)$$

$$\frac{dy}{dx} = \frac{(1+x^2)(1) - x\,(2x)}{(1+x^2)^2}$$

$$= \frac{1+x^2 - 2x^2}{(1+x^2)^2}$$

$$\frac{dy}{dx} = \frac{1-x^2}{(1+x^2)^2} \quad - \text{(ii)}$$

$$\frac{d^2y}{dx^2} = \frac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)2x}{(1+x^2)^4}$$

$$= \frac{(1+x^2)2x\left[-1-x^2-2+2x^2\right]}{(1+x^2)^4}$$

$$\frac{d^2y}{dx^2} = \frac{2x(x^2-3)}{(1+x^2)^3} \quad \text{— (ii)}$$

for increasing

$$\frac{dy}{dx} > 0$$

$$1 - x^2 > 0 \quad \text{[from (2)]}$$

$$-x^2 > -1$$

$$x^2 < 1$$

$$-1 < x < 1$$

Curve is increasing for $(-1, 1)$

for decreasing.

Illy Curve is decreasing for
$(-\infty, -1) \cup (1, \infty)$

7. Point of inflexion.

$$\frac{d^2y}{dx^2} = 0$$

$$2x(x^2 - 3) = 0 \quad [\text{from } (11)]$$

$$x = 0, \quad x^2 - 3 = 0$$

$x = 0$; $x = \pm\sqrt{3}$.

$x = 0, \sqrt{3}, -\sqrt{3}$.

$\frac{d^2y}{dx^2}$ changes signs at $x = 0, \sqrt{3}, -\sqrt{3}$.

$\therefore$ Point of inflexion at $x = 0, \sqrt{3}, -\sqrt{3}$

Points of inflexion are

$(0,0)$; $(\sqrt{3}, \sqrt{3}/4)$; $(-\sqrt{3}, -\sqrt{3}/4)$

Concave upward. $\frac{d^2y}{dx^2} > 0$

$x\,(x^2-3) > 0$ [from ②]

$x > 0$

$x^2 - 3 > 0$

$x^2 > 3$

$x > \sqrt{3}$.

$x < 0$

$x^2 - 3 < 0$

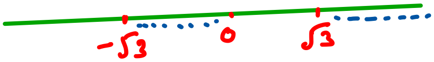
$x^2 < 3$

$x < \sqrt{3}$

$x > -\sqrt{3}$.

Concave upward for

$(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$



Illy Concave downward for

$$(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$$

& Maxima and Minima.

$$\frac{dy}{dx} = 0$$

$$1 - x^2 = 0$$

$$(1+x)(1-x) = 0$$

$$x = -1, +1$$

$\therefore$ tangent at $x=1$ and $x=-1$ are parallel to x- axis.

for $x=1$,

$$\frac{d^2y}{dx^2} = \frac{2x(x^2-3)}{(1+x^2)^3} \quad [\text{from (iii)}]$$

$$= \frac{2(1-3)}{(1+1)^3} = \frac{2(-2)}{8} = -1/2 < 0$$

$$\frac{d^2y}{dx^2} < 0$$

$\therefore$ Curve has Max value at $x=1$.

Maxima Point is $\left(1, \frac{1}{2}\right)$

for $x = -1$ $\quad \dfrac{d^2y}{dx^2} = \dfrac{2(-1)(1-3)}{(1+1)^3}$

$$= \frac{2(2)}{8} = \frac{1}{2} > 0$$

$$\frac{d^2y}{dx^2} > 0$$

$\therefore$ curve has Minima at $x = -1$

Minima is $(-1, -1/2)$

x	0	1	-1	2	-2	
y	0	$1/2$	$-1/2$	0.4	-0.4