Calculus II

# Asymptotes

Find all the asymptotes of the curve

$$x^3 - 2y^3 + xy(2x-y) + y(x-y) + 1 = 0$$

Sol.

$$x^3 - 2y^3 + 2x^2y - xy^2 + y(x-y) + 1 = 0$$

$$(x+y)(x-y)(x+2y) + y(x-y) + 1 = 0 \quad \text{--- (1)}$$

$x^3 - 2y^3 + 2x^2y - xy^2$

$x^2(x+2y) - y^2(x+2y)$

$(x^2-y^2)(x+2y)$

$(x-y)(x+y)(x+2y)$

$$x-y+\lim_{\substack{x\to\infty\\ y\to x}}\frac{y(x-y)+1}{(x+y)(x+2y)}=0$$

$$(x-y)+\lim_{x\to\infty}\frac{x(x-x)+1}{(x+x)(x+2x)}=0$$

$$(x-y)+\lim_{x\to\infty}\frac{0+1}{(2x)(3x)}=0$$

$$(x-y) + \lim_{x\to\infty} \frac{1}{6x^2} = 0$$

$$\boxed{x-y=0}$$

$$(x+y)(x-y)(x+2y) + y(x-y) + 1 = 0 \quad \text{— from ①}$$

$$(x+y) + \lim_{\substack{x\to\infty \\ y\to -x}} \frac{y(x-y)+1}{(x-y)(x+2y)} = 0$$

$$(x+y) + \lim_{x \to \infty} \frac{(-x)(x+x)+1}{(x+x)(x-2x)} = 0$$

$$(x+y) + \lim_{x \to \infty} \frac{-2x^2+1}{(2x)(-x)} = 0$$

$$(x+y) + \lim_{x \to \infty} \frac{-2x^2+1}{-2x^2} = 0$$

$$(x+y) + \lim_{x \to \infty} \left( \frac{-2x^2}{-2x^2} + \frac{1}{-2x^2} \right) = 0$$

$$(x+y) + \lim_{x\to\infty} 1 - \frac{1}{2x^2} = 0$$

$$(x+y) + 1 = 0$$

from ①

$$(x+y)(x-y)(x+2y) + y(x-y) + 1 = 0$$

$$(x+2y) + \lim_{\substack{x\to\infty \\ y\to -x/2}} \frac{y(x-y)+1}{(x+y)(x-y)} = 0$$

$$(x+2y) + \lim_{x\to\infty} \frac{\left(\frac{-x}{2}\right)\left(x+\frac{x}{2}\right)+1}{\left(x-\frac{x}{2}\right)\left(x+x/2\right)} = 0$$

$$(x+2y) + \lim_{x\to\infty} \frac{(-x/2)(3x/2)+1}{(x/2)(3x/2)} = 0$$

$$(x+2y) + \lim_{x\to\infty} \frac{-3x^2/4+1}{3x^2/4} = 0$$

$$(x+2y) + \lim_{x\to\infty} \frac{-3x^2/4}{3x^2/4} + \frac{4}{3x^2} = 0$$

$$(x+2y) + \lim_{x\to\infty} -1 + \frac{4}{3x^2} = 0$$

$$(x+2y) - 1 = 0$$

Asymptotes are

$$x - y = 0$$

$$x + y + 1 = 0$$

$$x + 2y - 1 = 0$$