THEORY OF EQUATIONS

Repeated Roots

Show that $x^3 - x^2 - x + 1 = 0$ has a repeated root and solve it.

Sol. Given eq. is

$$f(x) = x^3 - x^2 - x + 1$$

$$f'(x) = 3x^2 - 2x - 1$$

$$f'(x) = 0$$

$$3x^2 - 2x - 1 = 0$$

$$3x^2 - 3x + x - 1 = 0$$

$$3x(x-1) + 1(x-1) = 0$$

$$(3x+1)(x-1) = 0$$

$$x = 1, -1/3.$$

1 & $-1/3$ are roots of $f'(x)$

$f(1) = 1 - 1 - 1 + 1 = 0$

$\Rightarrow$ 1 is root of $f(x)$

$\therefore$ 1 is common root of $f(x)$ & $f'(x)$

$\Rightarrow$ 1 is a repeated root of $f(x)$

$f(x) = 1, 1, \alpha$

$\Rightarrow$ $(x-1)^2$ are factors of $f(x)$

$x^2 - 2x + 1$ divides $f(x)$

$$
\begin{array}{r}
x + 1 \\
x^2 - 2x + 1 \,\big)\, \overline{x^3 - x^2 - x + 1} \\
\underline{\cancel{x^3} - 2x^2 + x} \\
\cancel{x^2} - 2x + 1 \\
\underline{\cancel{x^2} - 2x + 1} \\
0
\end{array}
$$

The other root of $f(x)$ is given by

$x + 1 = 0$

$x = -1$

$\Rightarrow$ Roots of $f(x)$ are

$1, 1, -1$

✲ Show that $x^3 - 3x^2 - 4 = 0$ has no repeated root.

Sol. Given

$f(x) = x^3 - 3x^2 - 4$

$f'(x) = 3x^2 - 6x$

$f'(x) = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

0, 2 are roots of $f'(x)$

$x^3 - 3x^2 - 4$

$f(0) = -4 \neq 0$

$f(2) = 8 - 12 - 4 = -8 \neq 0$

$0, 2$ are not roots of $f(x)$

$\Rightarrow$ $f(x)$ and $f'(x)$ has no common root.

$\Rightarrow$ $f(x)$ has no repeated root.