

THEORY OF EQUATIONS

Repeated Roots

Show that $x^3 - x^2 - x + 1 = 0$ has a repeated root and solve it.

Sol.
=

Given eq. is

$$f(x) = x^3 - x^2 - x + 1$$

$$f'(x) = 3x^2 - 2x - 1$$



$$f'(x) = 0$$

$$3x^2 - 2x - 1 = 0$$

$$3x^2 - 3x + x - 1 = 0$$

$$3x(x-1) + 1(x-1) = 0$$

$$(3x+1)(x-1) = 0$$

$$x = 1, -1/3.$$

$$1 \text{ \& } -1/3$$

are roots of $f'(x)$



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$$f(1) = 1 - 1 - 1 + 1 = 0$$

$\Rightarrow 1$ is root of $f(x)$

$\therefore 1$ is common root of $f(x)$ & $f'(x)$

$\Rightarrow 1$ is a repeated root of $f(x)$

$$f(x) = (x-1)^2(x-d)$$

$\Rightarrow (x-1)^2$ are factors of $f(x)$

$x^2 - 2x + 1$ divides $f(x)$

$$\begin{array}{r} x^2 - 2x + 1 \overline{) x^3 - x^2 - x + 1} \\ \underline{-x^3 + x} \\ + x + 1 \end{array}$$

$$\begin{array}{r} x^2 - 2x + 1 \overline{) x^2 - 2x + 1} \\ \underline{-x^2 + 1} \\ + 1 \end{array}$$

0

The other root of $f(x)$ is given by
 $x + 1 = 0$

$$x = -1$$

\Rightarrow Roots of $f(x)$ are

$$1, 1, -1$$

~~*~~ Show that $x^3 - 3x^2 - 4 = 0$ has no repeated root.

Sol.

Given

$$f(x) = x^3 - 3x^2 - 4$$

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0$$

$$3x^2 - 6x = 0$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, 2$$

0, 2 are roots of $f'(x)$

$$f(0) = -4 \neq 0$$

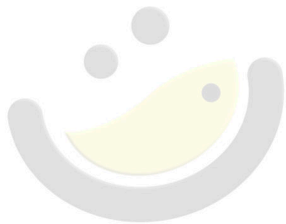
$$f(2) = 8 - 12 - 4 = -8 \neq 0$$

$$x^3 - 3x^2 - 4$$

0, 2 are not roots of $f(x)$

$\Rightarrow f(x)$ and $f'(x)$ has no common root.

$\Rightarrow f(x)$ has no repeated root.



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