

# THEORY OF EQUATIONS

## Repeated Roots

Solve the equation

$$x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$$

Given that it has a repeated root.

Sol.  
=

$$f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$$

$$f'(x) = 4x^3 + 12x^2 - 12x - 36$$

Given that

$f(x)$  has repeated root

$\therefore f(x)$  and  $f'(x)$  has at least  
One Common Root.

$$\therefore x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$$

and  $4x^3 + 12x^2 - 12x - 36 = 0$  has  
 $x^3 + 3x^2 - 3x - 9 = 0$   
Common Root.

$$x^2 + 3x^2 - 3x - 9 \overline{)x^4 + 4x^3 - 6x^2 - 36x - 27} \quad x+1$$

$$\underline{-x^4 + 3x^3 - 3x^2 - 9x}$$

$$x^3 - 3x^2 - 27x - 27$$

$$\underline{-x^3 + 3x^2 - 3x - 9}$$

$$-6x^2 - 24x - 18 \overline{)x^3 + 3x^2 - 3x - 9} \quad x-1$$

$$x^2 + 4x + 3$$

$$\underline{-x^3 + 4x^2 + 3x}$$

$$-x^2 - 6x - 9$$

$$\underline{-x^2 - 4x - 3}$$

$$x+1$$

$$-2x - 6 \overline{)x^2 + 4x + 3}$$

$$x+3$$

$$\underline{-x^2 - 3x}$$

$$x+3$$

$$\underline{x+3}$$

$$\underline{x}$$

$\therefore$  g.c.d of  $f(x) + f'(x)$  is

$$\frac{x+3}{=}$$

Put  $g.c.d = 0$

$$x+3 = 0$$

$$x = -3$$

$\therefore$  Common Root of  $f(x) + f'(x)$   
is  $-3$ .

Repeated Root of  $f(x)$  is  $-3, -3$

$(x+3)$ ,  $(x+3)$  are factors of  $f(x)$

By synthetic division.

The diagram illustrates the use of synthetic division twice to factor a polynomial. It features two horizontal lines for division. The first division has a divisor of  $x + 3$  (with a root of  $-3$ ) and a dividend of  $x^4 - 6x^3 - 36x^2 - 27x$ . The second division has a divisor of  $x + 3$  (with a root of  $-3$ ) and a dividend of  $x^3 - 9x^2 - 9x$ . The remainders from both divisions are zero, confirming that  $(x + 3)$  is a factor of the original polynomial.

$-3$	$ $	$1$	$4$	$-6$	$-36$	$-27$
			$-3$	$-3$	$27$	$27$
				$-9$	$-9$	$0$
$-3$	$ $		$-3$	$6$	$9$	
				$-3$	$0$	

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1, 3.$$

∴ Roots of eq.  $f(x)$  are

-1, 3, -3, -3 aly.