THEORY OF EQUATIONS

Repeated Roots

Solve the equation

$$x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$$

Given that it has a repeated root.

Sol.

$$f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$$

$$f'(x) = 4x^3 + 12x^2 - 12x - 36$$

Given that

$f(x)$ has repeated root

$\therefore$ $f(x)$ and $f'(x)$ has at least one Common Root.

$\therefore$ $x^4 + 4x^3 - 6x^2 - 36x - 27 = 0$

and $4x^3 + 12x^2 - 12x - 36 = 0$ has

$x^3 + 3x^2 - 3x - 9 = 0$

Common Root.

$x^3+3x^2-3x-9 \,\overline{\smash{)}\, \cancel{x^4}+4x^3-6x^2-36x-27}$ $\quad x+1$

$-\cancel{x^4} \mp 3x^3 \pm 3x^2 \pm 9x$

$\cancel{x^3}-3x^2-27x-27$

$-\cancel{x^3} \mp 3x^2 \pm 3x \pm 9$

$-6x^2-24x-18 \,\overline{\smash{)}\, \cancel{x^3}+3x^2-3x-9}$ $\quad x-1$

x^2+4x+3

$-\cancel{x^3} - 4x^2 - 3x$

$-\cancel{x^2}-6x-9$

$+\cancel{x^2} \mp 4x \mp 3$

$-2x-6 \,\overline{\smash{)}\, \cancel{x^2}+4x+3}$ $\quad x+1$

$x+3$

$-\cancel{x^2} - 3x$

$x+3$

$x+3$

$\cancel{x}$

$\therefore$ g.c.d of $f(x)$ & $f'(x)$ is

$$x+3.$$

Put g.c.d = 0

$$x+3=0$$

$$x=-3$$

$\therefore$ Common Root of $f(x)$ & $f'(x)$ is -3.

Repeated Root of $f(x)$ is $-3, -3$

$(x+3)$, $(x+3)$ are factors of $f(x)$

By synthetic division.

$$
\begin{array}{r|rrrrr}
-3 & 1 & 4 & -6 & -36 & -27 \\
 & & -3 & -3 & 27 & 27 \\
\hline
 & 1 & 1 & -9 & -9 & 0 \\
-3 & & -3 & 6 & 9 & \\
\hline
 & 1 & -2 & -3 & 0 &
\end{array}
$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + 1(x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$x = -1, 3.$$

$\therefore$ Roots of eq. $f(x)$ are

$-1, 3, -3, -3$ Ans.