Concavity and Convexity

Show that the points of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lies on the line $3x + a = 4b$.

Sol

$$y^2 = (x-a)^2(x-b)$$

$$y = \pm (x-a)\sqrt{x-b}$$

Let $y = (x-a)\sqrt{x-b}$

$$\frac{dy}{dx} = (x-a)\frac{1}{2\sqrt{x-b}} + \sqrt{x-b}\,(1)$$

$$= \frac{(x-a) + 2(x-b)}{2\sqrt{x-b}}$$

$$= \frac{x - a + 2x - 2b}{2\sqrt{x-b}}$$

$$\frac{dy}{dx} = \frac{3x - a - 2b}{2\sqrt{x-b}}$$

$$\frac{d^2y}{dx^2} = \frac{2\sqrt{x-b}\,(3) - (3x - a - 2b)\,\frac{2}{2\sqrt{x-b}}}{4(x-b)}$$

$$= \frac{3x2\,(x-b) - 3x + a + 2b}{4(x-b)^{3/2}}$$

$$= \frac{6x - 6b - 3x + a + 2b}{4(x-b)^{3/2}}$$

$$\frac{d^2y}{dx^2} = \frac{3x + a - 4b}{4(x-b)^{3/2}} \quad \text{— (1)}$$

for point of inflexion

$$\frac{d^2y}{dx^2} = 0$$

$$3x + a - 4b = 0 \qquad [\text{from } (1)]$$

$$3x + a = 4b$$

Similarly if we take $y = -(x-a)\sqrt{x-b}$. We will find point of inflexion on $3x + a = 4b$

Hence Proved