Concavity and Convexity

Determine a and b so that the curve $y = ax^3 + 3bx^2$ has a point of Inflexion at $(-1, 2)$

Sol:

$$y = ax^3 + 3bx^2 \quad \text{— (1)}$$

$$\frac{dy}{dx} = 3ax^2 + 6bx$$

$$\frac{d^2y}{dx^2} = 6ax + 6b. \quad -(II)$$

Given Point of inflexion is $P(-1, 2)$

Point of inflexion lies on the curve (I)

$$\therefore y = ax^3 + 3bx^2$$

$$2 = a(-1)^3 + 3b(-1)^2$$

$$2 = -a + 3b \quad -(III)$$

Now for point of Inflexion

$$\frac{d^2y}{dx^2} = 0 \quad \text{at point } (-1, 2)$$

$$\Rightarrow 6ax + 6b = 0 \quad [\text{from } (1)]$$

$$\Rightarrow ax + b = 0$$

$$\Rightarrow a(-1) + b = 0$$

$$\Rightarrow -a + b = 0$$

$$a = b.$$

Put $a = b$ in (ii)

$$2 = -a + 3a.$$

$$2 = 2a$$

$$a = 1 \Rightarrow b = 1$$

$a = b = 1$. Ans.