

# Calculus II

## Concavity and Convexity

Show that every point in which the curve  $y = c \sin \frac{x}{a}$  meets  $x$ -axis is a point of inflexion of the curve.

Sol.

$$y = c \sin \frac{x}{a} \quad \text{--- (1)}$$

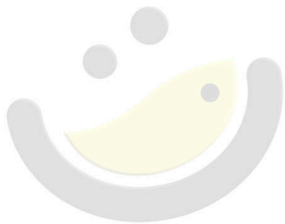
$$\frac{dy}{dx} = c \cos \frac{x}{a} \quad \left(\frac{1}{a}\right)$$

$$= \frac{c}{a} \cos \frac{x}{a}$$

$$\frac{d^2y}{dx^2} = -\frac{c}{a} \sin \frac{x}{a} \cdot \frac{1}{a}$$

$$\frac{d^2y}{dx^2} = -\frac{c}{a^2} \sin \frac{x}{a} \quad \text{--- (2)}$$

$$\frac{d^3y}{dx^3} = \frac{-c}{a^3} \cos \frac{x}{a} \quad \text{--- (3)}$$



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The curve meet the  $x$ -axis where

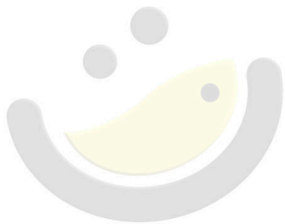
from  $y = 0$

$$c \sin \frac{x}{a} = 0$$

$$\sin \frac{x}{a} = 0$$

$$\Rightarrow \frac{x}{a} = n\pi$$

$$\Rightarrow \underline{\underline{x = an\pi}}$$



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Put  $x = a n \pi$  in  $\frac{d^2y}{dx^2}$

$$-\frac{c}{a^2} \sin \frac{x}{a} = -\frac{c}{a^2} \sin \frac{a n \pi}{a}$$

$$= -\frac{c}{a^2} \sin n \pi = 0$$

$$\frac{d^2y}{dx^2} = 0$$

def

$$[\because \sin n \pi = 0]$$


$$\text{Put } x = an\pi \text{ in (3)}$$

$$\frac{d^3y}{dx^3} = \frac{-c}{a^3} \cos \frac{an\pi}{a}$$

$$= \frac{-c}{a^3} \cos n\pi$$

$$= \frac{-c}{a^3} (-1)^n = (-1)^{n+1} \frac{c}{a^3} \neq 0$$

$$\frac{d^3y}{dx^3} \neq 0 \quad \text{at } x = 2n\pi.$$

Every point where the curve meet  
x-axis is point of inflexion.



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