Calculus II

Concavity and Convexity

Show that every point in which the curve $y = c\sin\frac{x}{a}$ meets x-axis is a point of inflexion of the curve.

Sol.

$$y = c\sin\frac{x}{a} \quad —(1)$$

$$\frac{dy}{dx} = c\cos\frac{x}{a}\left(\frac{1}{a}\right)$$

$$= \frac{c}{a} \cos \frac{x}{a}$$

$$\frac{d^2y}{dx^2} = -\frac{c}{a} \sin \frac{x}{a} \cdot \frac{1}{a}.$$

$$\frac{d^2y}{dx^2} = -\frac{c}{a^2} \sin \frac{x}{a} \quad -\text{②}$$

$$\frac{d^3y}{dx^3} = -\frac{c}{a^3} \cos \frac{x}{a}. \quad -\text{③}$$

The curve meet the x-axis where

$$y = 0$$

from ①

$$c \sin\frac{x}{a} = 0$$

$$\sin\frac{x}{a} = 0$$

$$\Rightarrow \frac{x}{a} = n\pi$$

$$\Rightarrow \underline{\underline{x = an\pi.}}$$

Put $x = an\pi$ in $\dfrac{d^2y}{dx^2}$

$$-\frac{c}{a^2}\sin\frac{x}{a} = -\frac{c}{a^2}\sin\frac{an\pi}{a}$$

$$= -\frac{c}{a^2}\sin n\pi = 0$$

$\left[\because \underbrace{\sin n\pi}_{=0}\right]$

$\dfrac{d^2y}{dx^2} = 0$ def

Put $x = an\pi$ in (3)

$$\frac{d^3y}{dx^3} = -\frac{c}{a^3} \cos \frac{an\pi}{a}$$

$$= -\frac{c}{a^3} \cos n\pi$$

$$= -\frac{c}{a^3} (-1)^n = (-1)^{n+1} \frac{c}{a^3} \neq 0$$

$$\frac{d^3y}{dx^3} \neq 0 \quad \text{at} \quad x = 2n\pi.$$

Every point where the Curve meet x-axis is point of inflexion.