Asymptotes

find the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

Sol. Given curve is

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

$y = mx + c$

$y = m_1x + c_1$

$y = m_2x + c_2$

$y = m_3x + c_3$

$x = 1 \qquad y = m.$

$$Q_3(m) = 1 + 2m - m^2 - 2m^3$$

$$Q_2(m) = 4m^2 + 2m$$

$$Q_1(m) = y - 1$$

$$Q_3(m) = 0$$

$$1 + 2m - m^2 - 2m^3 = 0$$

$$2m^3 + m^2 - 2m - 1 = 0$$

$$m^2(2m+1) - 1(2m+1) = 0$$

$$(m^2-1)(2m+1) = 0$$

$$(m+1)(m-1)(2m+1) = 0$$

$$m = 1, -1, -1/2.$$

$$\Rightarrow m_1 = 1 \; ; \; m_2 = -1 \; ; \; m_3 = -1/2$$

$$C = -\frac{q_2(m)}{q_3'(m)} = \frac{-(4m^2+2m)}{-6m^2-2m+2}$$

$$= \frac{-2(2m^2+m)}{-2(3m^2+m-1)}$$

$$c = \frac{2m^2+m}{3m^2+m-1} \quad \text{— (1)}$$

for $m_1 = 1$

$$c_1 = \frac{2+1}{3+1-1} = 1 \qquad [\text{from (1)}]$$

$c_1 = 1$. for $m_1 = 1$

$y = m_1 x + c$

$y = x + 1$

for $m_2 = -1$

$c_2 = 1$

$y = m_2 x + c_2$

$= -x + 1$

$$y + x - 1 = 0$$

for $m_3 = -1/2$

$$c_3 = \frac{2(-1/2)^2 + (-1/2)}{3(-1/2)^2 + (-1/2) - 1}$$

$$= \frac{2 \times 1/4 - 1/2}{3 \times 1/4 - 1/2 - 1} = 0$$

$$c_3 = 0$$

$$y = m_3 x + c_3$$

$$y = -1/2\, x + 0$$

$$y = -1/2\, x.$$