

Calculus II

Asymptotes

find the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

Sol.
=

Given curve is

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

$$y = mx + c$$

$$y = m_1x + c_1$$

$$y = m_2x + c_2$$

$$y = m_3x + c_3$$

$$x = 1 \quad y = m.$$

$$Q_3(m) = 1 + 2m - m^2 - 2m^3$$

$$Q_2(m) = 4m^2 + 2m$$

$$Q_1(m) = y - 1$$

$$Q_3(m) = 0$$

$$1 + 2m - m^2 - 2m^3 = 0$$

$$2m^3 + m^2 - 2m - 1 = 0$$

$$m^2 (2m+1) - 1(2m+1) = 0$$

$$(m^2 - 1)(2m+1) = 0$$

$$(m+1)(m-1)(2m+1) = 0$$

$$m = 1, -1, -\frac{1}{2}$$

$$\Rightarrow m_1 = 1; m_2 = -1; m_3 = -\frac{1}{2}$$

$$C = -\frac{\phi_2(m)}{\phi_3'(m)} = \frac{-(4m^2 + 2m)}{-6m^2 - 2m + 2}$$



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$$= \frac{-2(2m^2 + m)}{-2(3m^2 + m - 1)}$$

$$C = \frac{2m^2 + m}{3m^2 + m - 1} \quad \text{--- (1)}$$

for $m_1 = 1$

$$C_1 = \frac{2 + 1}{3 + 1 - 1} = 1$$

[from (1)]

$$c_1 = 1. \text{ for } m_1 = 1$$

$$y = m_1 x + c_1$$

$$y = x + 1$$

for $m_2 = -1$

$$c_2 = 1$$

$$\begin{aligned} y &= m_2 x + c_2 \\ &= -x + 1 \end{aligned}$$

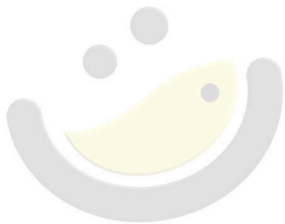
$$y + x - 1 = 0$$

for $m_2 = -\frac{1}{2}$

$$C_3 = \frac{2 \left(-\frac{1}{2} \right)^2 + \left(-\frac{1}{2} \right)}{3 \left(-\frac{1}{2} \right)^2 + \left(-\frac{1}{2} \right) - 1}$$

$$= \frac{2 \times \frac{1}{4} - \frac{1}{2}}{3 \times \frac{1}{4} - \frac{1}{2} - 1} = 0$$

$$C_3 = 0$$

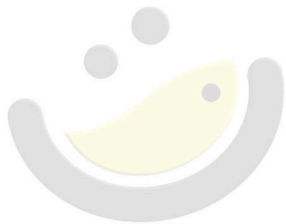


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$$y = m_3 x + c_3$$

$$y = -1/2 x + 0$$

$$y = -1/2 x.$$



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