Multiple Points

Determine the position and nature of double point on the curve

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$$

Sol.

$$f(x, y) = x^3 - y^2 - 7x^2 + 4y + 15x - 13.$$

$$\frac{\partial f}{\partial x} = 3x^2 - 14x + 15$$

$$\frac{\partial f}{\partial y} = -2y + 4.$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

$$\frac{\partial^2 f}{\partial x^2} = 6x - 14 \quad — (1)$$

$$\frac{\partial^2 f}{\partial y^2} = -2. \quad — (11)$$

$$\frac{\partial f}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$$

$$3x^2 - 14x + 15 = 0 \qquad -2y + 4 = 0$$

$$3x^2 - 9x - 5x + 15 = 0 \qquad y = 2.$$

$$3x(x-3) - 5(x-3) = 0$$

$$(x-3)(3x-5) = 0$$

$$x = 3, 5/3.$$

$\therefore$ Possible double points are

$$(3,2)\ ;\ (5/3, 2)$$

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$$

$$27 - 4 - 63 + 8 + 45 - 13 = 0$$

$$80 - 80 = 0$$

$$0 = 0$$

$(3,2)$ satifies the given curve $f(x,y)$

Illly we can check for $(5/3, 2)$

$(5/3, 2)$ does not satisfy the curve

$\therefore$ Curve has only one double point $(3, 2)$

$$D = \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} = 0 - 4(-2)$$

$$= 8 > 0$$

$D > 0$ point is node.