

Calculus II

Asymptotes

Show that asymptote of the cubic

Curve

$$x^3 - xy^2 - 2xy + 2x - y - 1 = 0 \text{ cut the}$$

Curve in at most three points which

lies on the line $3x - y - 1 = 0$

Sol.

Given Curve is

$$x^3 - xy^2 - 2xy + 2x - y - 1 = 0$$

$$x = 1 \quad y = m.$$

$$Q_3(m) = 1 - m^2$$

$$Q_2(m) = -2m.$$

$$Q_1(m) = 2 - m.$$

$$\text{Put } Q_3(m) = 0$$

$$1 - m^2 = 0$$

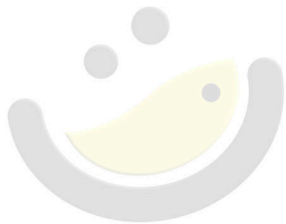
$$(1 - m)(1 + m) = 0$$

$$m = -1, 1.$$

$$C = - \frac{q_2(m)}{q_3'(m)}$$

$$= - \frac{(-2m)}{-2m} = -1.$$

$$C = -1.$$



OMG! MATHS }
The poetry of logical ideas.

Asymptotes are

$$y = mx + c$$

for $m = -1$

$$y = -x - 1$$

$$y + x + 1 = 0$$

for $m = 1$.

$$y = x - 1$$

$$x - y - 1 = 0$$

Also Asymptote parallel to y -axis is $x = 0$

Asymptotes are

$$x = 0 \quad \text{--- (1)}$$

$$x + y + 1 = 0 \quad \text{--- (2)}$$

$$x - y - 1 = 0 \quad \text{--- (3)}$$

Multiply (1), (2) & (3)

$$x(x + y + 1)(x - y - 1) =$$

$$(x^2 + xy + x)(x - y - 1)$$

$$= x^3 - \cancel{x^2y} - \cancel{x^2} + \cancel{x^2y} - xy^2 - xy + \cancel{x^2} - xy - x$$

$$= x^3 - xy^2 - 2xy - x - \textcircled{IV}$$

subtract \textcircled{IV} from given curve.

$$\cancel{x^3} - \cancel{xy^2} - \cancel{2xy} + 2x - y - 1 - \cancel{x^3} + \cancel{xy^2} + \cancel{2xy} + x$$

$$3x - y - 1 = 0$$

\Rightarrow Asymptotes cut the curve in points
which lies on $3x - y - 1 = 0$

No. of points are $n(n-2)$
 $3(3-2) = 3.$

\Leftarrow Asymptotes cut the curve in three
points which lies on.

$3x - y - 1 = 0$ Hence Proved.