Concavity and Convexity

Find the Values of x for which

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7$$

is Concave upward or downward.

Also determine the point of inflection.

Sol.

$$y = x^4 - 6x^3 + 12x^2 + 5x + 7 \quad -(1)$$

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 24x + 5$$

$$\frac{d^2y}{dx^2} = 12x^2 - 36x + 24 \quad \text{— (ii)}$$

$$\frac{d^3y}{dx^3} = 24x - 36 \quad \text{— (iii)}$$

for Concave upward

$$\frac{d^2y}{dx^2} > 0$$

$\Rightarrow \quad 12x^2 - 36x + 24 > 0$ [from (11)]

$\Rightarrow \quad x^2 - 3x + 2 > 0$

$\Rightarrow \quad x^2 - 3x > -2$

$\Rightarrow \quad x^2 - 3x + \frac{9}{4} - \frac{9}{4} > -2$

$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 > -2 + 9/4$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 > \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad \left| x - \frac{3}{2} \right| > \frac{1}{2}$$

$$\Rightarrow \quad \frac{-1}{2} > x - \frac{3}{2} > \frac{1}{2}$$

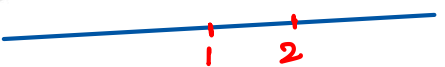
$$\Rightarrow \quad \frac{-1}{2} + \frac{3}{2} > x - \frac{3}{2} + \frac{3}{2} > \frac{1}{2} + \frac{3}{2}$$

$\Rightarrow \quad 1 > x > 2$

$\Rightarrow \quad x < 1 \ \& \ x > 2$

$\therefore$ Curve is Concave upward for

$(-\infty, 1) \cup (2, \infty)$



for Concave downward.

$$\frac{d^2y}{dx^2} < 0$$

$$12x^2 - 36x + 24 < 0 \quad [\text{from (ii)}]$$

$$\Rightarrow \left|x - \tfrac{3}{2}\right| < \tfrac{1}{2}$$

$$\Rightarrow -\frac{1}{2} < x - \tfrac{3}{2} < \tfrac{1}{2}$$

$\Rightarrow \quad 1 < x < 2$

Curve is Concave downward for

$(1, 2)$

$$\frac{d^2y}{dx^2} = 0$$

$12x^2 - 36x + 24 = 0$ [from (ii)]

$x^2 - 3x + 2 = 0$

$$x^2 - 2x - x + 2 = 0$$

$$x(x-2) - 1(x-2) = 0$$

$$x = 1, 2.$$

$$\frac{d^3y}{dx^3} = 24x - 36 \qquad \text{[from (iii)]}$$

for $x = 1$ $\quad = 24(1) - 36 = -12 \neq 0$

for $x = 2$ $\quad \frac{d^3y}{dx^3} = 24(2) - 36 = 12 \neq 0$

$\Rightarrow x = 1, 2$ are point of inflexion.

for $x=1$ $\quad y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$y = 1 - 6 + 12 + 5 + 7 = 19.$

for $x=2$ $\quad y = (2)^4 - 6(2)^3 + 12(2)^2 + 5\times 2 + 7$

$= 16 - 6 \times 8 + 12 \times 4 + 10 + 7$

$= 16 - 48 + 48 + 10 + 7 = 33.$

$\therefore$ Point of inflexions are

$(1, 19)$ & $(2, 33)$ Ans.