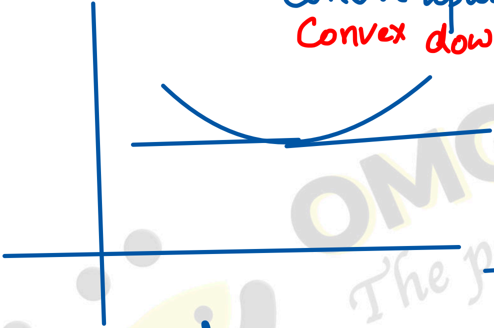
Calculus II

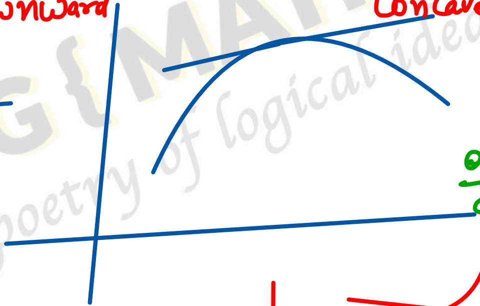
Concavity and Convexity

Concave upward
Convex downward

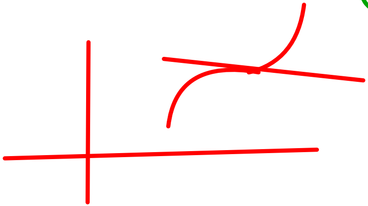


$$\frac{d^2y}{dx^2} > 0 \quad (a,b)$$

Convex. upward.
Concave downward



$$\frac{d^2y}{dx^2} < 0 \quad (a,b)$$



$y = ?$

$\frac{dy}{dx} = ?$

$\frac{d^2y}{dx^2}$

$\frac{d^3y}{dx^3} \neq 0$. exists

Point of inflexion

Prove that the curve $y = e^x$ is concave upward for all $x \in R$.

Sol.

$$y = e^x$$

$$\frac{dy}{dx} = e^x$$

$$\frac{d^2y}{dx^2} = e^x > 0 \quad x \in R$$

$$\frac{d^2y}{dx^2} > 0 \quad \forall\ x \in \mathbb{R}.$$

$\therefore$ The curve is concave upward.

Prove that the curve $y = \log x$ is everywhere concave downward for $x > 0$

Sol.

$$y = \log x$$

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{x^2} < 0$$

$$\frac{d^2y}{dx^2} < 0 \quad \text{for } x > 0$$

$\therefore$ The Curve is Convex (Concave downward)

Show that origin is the point of inflexion of the curve $y = x^{4/3}$

Sol:

$$y = x^{1/3}$$

$$\frac{dy}{dx} = \frac{1}{3} x^{-2/3}$$

$$\frac{d^2y}{dx^2} = \frac{1}{3}\left(-\frac{2}{3}\right)\left(x^{-5/3}\right)$$

$$\frac{d^2y}{dx^2} = \frac{-2}{9} x^{-5/3}$$

$\frac{d^2y}{dx^2}$ does not exist at $x = 0$

for $x > 0$ $\qquad \frac{d^2y}{dx^2} < 0$

$x < 0$ $\qquad \frac{d^2y}{dx^2} > 0$

$\therefore$ O is Point of inflexion

$$y = x^{1/3}$$

$$y = 0$$

$(0, 0)$ Origin is a point of inflexion of the curve $y = x^{1/3}$