THEORY OF EQUATIONS

Common Roots

Find the common roots of equations

$$x^4 - 7x^2 + 1 = 0$$

$$x^3 - 2x^2 - 2x + 1 = 0$$

Solve them completely.

Sol

$$x^4 - 7x^2 + 1 = 0 \quad -(1)$$

$$x^3 - 2x^2 - 2x + 1 = 0 \quad -(11)$$

$x^3 - 2x^2 - 2x + 1$) $x^4 - 7x^2 + 1$ ($x + 2$

$-(x^4 - 2x^3 - 2x^2 + x)$

$2x^3 - 5x^2 - x + 1$

$-(2x^3 - 4x^2 - 4x + 2)$

$-x^2 + 3x - 1$) $x^3 - 2x^2 - 2x + 1$ ($-x - 1$

$-(x^3 - 3x^2 + x)$

$x^2 - 3x + 1$

$-(x^2 - 3x + 1)$

x

g. c. d. of (I) & (II) is

$x^2 - 3x + 1$ — (III)

$\therefore$ Common Roots of (I) & (II) are given by (III)

$$x^2 - 3x + 1 = 0$$

$ax^2 + bx + c = 0$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(1)}}{2}$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$\therefore$ Common Roots are $\frac{3 \pm \sqrt{5}}{2}$ — (IV)

e). ① is $x^4 - 7x^2 + 1 = 0$

g.c.d divides ①

$$\Rightarrow x^2 - 3x + 1 \;\overline{)\; x^4 - 7x^2 + 1 \;}\; x^2 + 3x + 1$$

$$-\cancel{x^4} \mp 3x^3 \pm x^2$$

$$\cancel{3x^3} - 8x^2 + 1$$

$$-\cancel{3x^3} \mp 9x^2 \pm 3x$$

$$x^2 - 3x + 1$$

$$x^2 - 3x + 1$$

$$\times$$

Other roots of eq (i) are given by

$x^2 + 3x + 1 = 0$ $\quad ax^2+bx+c=0$

$$x = \frac{-3 \pm \sqrt{9-4(1)(1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{5}}{2} \quad -(v)$$

∴ from (iv) & (v)

Roots of eq. (i) are

$$\frac{3 \pm \sqrt{5}}{2}, \quad \frac{-3 \pm \sqrt{5}}{2}$$

eq. (ii) is

$$x^3 - 2x^2 - 2x + 1 = 0$$

H.C.F divides (ii)

$$x^2 - 3x + 1 \;\overline{)\; x^3 - 2x^2 - 2x + 1 \;}\; x + 1$$

$$-\;(x^3 - 3x^2 + x)$$

$$x^2 - 3x + 1$$

$$-\;(x^2 - 3x + 1)$$

$$0$$

Other roots of eq (II) are given by

$$x + 1 = 0$$

$$x = -1. \quad - (VI)$$

Roots of eq (i) are

$$\frac{3 \pm \sqrt{5}}{2}, \quad -1$$

from (iv) & (vi)