

# THEORY OF EQUATIONS

## Common Roots

Find the Common roots of equations

$$x^4 - 7x^2 + 1 = 0$$

$$x^3 - 2x^2 - 2x + 1 = 0$$

Solve them completely.

Sol

$$x^4 - 7x^2 + 1 = 0 \quad \text{--- (i)}$$

$$x^3 - 2x^2 - 2x + 1 = 0 \quad \text{--- (ii)}$$

$$x^3 - 2x^2 - 2x + 1 \overline{) x^4 - 7x^2 + 1} \quad x+2$$

$$\begin{array}{r} x^4 \\ - x^4 \\ \hline + 2x^3 - 2x^2 + x \end{array}$$

$$2x^3 - 5x^2 - x + 1$$

$$\begin{array}{r} 2x^3 \\ - 2x^3 \\ \hline + 4x^2 - 4x + 2 \end{array} \quad -x-1$$

$$-x^2 + 3x - 1 \overline{) x^3 - 2x^2 - 2x + 1}$$

$$\begin{array}{r} x^3 \\ - x^3 \\ \hline + 3x^2 + x \end{array}$$

$$x^2 - 3x + 1$$

$$\begin{array}{r} x^2 - 3x + 1 \\ - x^2 + 3x - 1 \\ \hline x \end{array}$$

x

g. c. d. of (i) & (ii) is

$$x^2 - 3x + 1 \quad - \quad \text{(iii)}$$



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$\therefore$  Common roots of (i) & (ii) are given by (iii)

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{9 - 4(1)(1)}}{2}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$\therefore$  Common roots are  $\frac{3 \pm \sqrt{5}}{2}$  — (iv)

e). ① is  $x^4 - 7x^2 + 1 = 0$

g.c.d divides ①

$\Rightarrow x^2 - 3x + 1$

$$\begin{array}{r} x^2 + 3x + 1 \\ \hline x^4 \quad - \quad 7x^2 \quad + \quad 1 \\ - \quad x^4 \quad - \quad 3x^3 \quad + \quad x^2 \\ \hline \end{array}$$

$$\begin{array}{r} 3x^3 - 8x^2 + 1 \\ - \quad 3x^3 - 9x^2 + 3x \\ \hline \end{array}$$

$$\begin{array}{r} x^2 - 3x + 1 \\ x^2 - 3x + 1 \\ \hline x \end{array}$$



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Other roots of eq (i) are given by

$$x^2 + 3x + 1 = 0$$

$$ax^2 + bx + c = 0$$

$$x_2 = \frac{-3 \pm \sqrt{9 - 4(1)(1)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{5}}{2} \quad \text{--- (v)}$$

$\therefore$  from (iv) & (v)

Roots of eq. ① are

$$\frac{3 \pm \sqrt{5}}{2}, \quad \frac{-3 \pm \sqrt{5}}{2}$$

eq. ① is

$$x^3 - 2x^2 - 2x + 1 = 0$$

b.f.c.d divides ①

$$\begin{array}{r}
 x^2 - 3x + 1 \quad \sqrt{\phantom{x^2 - 3x + 1}} \\
 \phantom{x^2 - 3x + 1} \quad x^2 - 2x^2 - 2x + 1 \quad \color{red}{x+1} \\
 \phantom{x^2 - 3x + 1} \quad \underline{x^2 - 3x^2 + x} \\
 \phantom{x^2 - 3x + 1} \quad \phantom{x^2 - 3x^2} + x^2 - 3x + 1 \\
 \phantom{x^2 - 3x + 1} \quad \phantom{x^2 - 3x^2} \underline{x^2 - 3x + 1} \\
 \phantom{x^2 - 3x + 1} \quad \phantom{x^2 - 3x^2} \phantom{x^2 - 3x + 1} \quad \underline{x}
 \end{array}$$

Other roots of eq (i) are given by

$$x + 1 = 0$$

$$x = -1. \quad \text{--- (vi)}$$

Roots of eq (i) are

$$\frac{3 \pm \sqrt{5}}{2}$$

1 -1 from (iv) & (v)

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