THEORY OF EQUATIONS

Form an equation of lowest degree with Rational Coeff. if one of its roots is $\sqrt{2}+\sqrt{5}$.

Sol. Let $f(x)=0$ is Required eq.

Now Given Root of $f(x)$ is $\sqrt{2}+\sqrt{5}$

In an eq. with Rational coeff. irrational roots occur in Conjugate pairs.

$\Rightarrow \sqrt{2}-\sqrt{5}, -\sqrt{2}+\sqrt{5}, -\sqrt{2}-\sqrt{5}$ are roots of $f(x)$

$\Rightarrow \left(x-(\sqrt{2}+\sqrt{5})\right), \left(x-(\sqrt{2}-\sqrt{5})\right),$

$\left(x-(-\sqrt{2}+\sqrt{5})\right)$ $\left(x-(-\sqrt{2}-\sqrt{5})\right)$ are factor of $f(x)$

$$\Rightarrow f(x) = \left(x - (\sqrt{2}+\sqrt{5})\right)\left(x - (\sqrt{2}-\sqrt{5})\right)$$

$$\left(x - (-\sqrt{2}+\sqrt{5})\right)\left(x - (-\sqrt{2}-\sqrt{5})\right)$$

$$= \left[(x-\sqrt{2}) - \sqrt{5}\right]\left[(x-\sqrt{2}) + \sqrt{5}\right]$$

$$\left[(x+\sqrt{2}) - \sqrt{5}\right]\left[(x+\sqrt{2}) + \sqrt{5}\right]$$

$$= \left[(x-\sqrt{2})^2 - 5\right]\left[(x+\sqrt{2})^2 - 5\right]$$

$$= (x^2 + 2 - 2\sqrt{2}\,x - 5)(x^2 + 2 + 2\sqrt{2}\,x - 5)$$

$$= ((x^2 - 3) - 2\sqrt{2}\,x)((x^2 - 3) + 2\sqrt{2}\,x)$$

$$= (x^2 - 3)^2 - (2\sqrt{2}\,x)^2$$

$$= x^4 + 9 - 6x^2 - 8x^2$$

$$= x^4 - 14x^2 + 9 = 0$$ Ans.