

THEORY OF EQUATIONS

Construct a polynomial equation over rationals of degree 4 whose roots are $\sqrt{3}$ and $1+2i$.

Sol.

Let the required eq. is $f(x)=0$

The given roots are $\sqrt{3}, 1+2i$ — (1)

In an eq. with rational coeff., irrational roots occur in conjugate pairs.

$\Rightarrow -\sqrt{3}$ is also root of $f(x)$ — (i)

In an eq. with real coeff, imaginary root occur in conjugate pairs.

$\Rightarrow 1 - 2i$ is also root of $f(x) = 0$ — (ii)

from (i), (ii) and (iii)

Roots of $f(x)$ are

$$\sqrt{3}, -\sqrt{3}, 1+2i, 1-2i$$

$$\Rightarrow (x - \sqrt{3}), (x + \sqrt{3}), (x - (1+2i))$$

$(x - (1-2i))$ are factors

of $f(x)$

\Rightarrow Required eq. is

$$f(x) = (x - \sqrt{3})(x + \sqrt{3})(x - (1+2i))(x - (1-2i))$$

$$= (x^2 - 3)[(x-1) - 2i][(x-1) + 2i]$$

$$= (x^2 - 3)((x-1)^2 - (2i)^2)$$

$$= (x^2 - 3)(x^2 + 1 - 2x + 4) \quad [\because i^2 = -1]$$

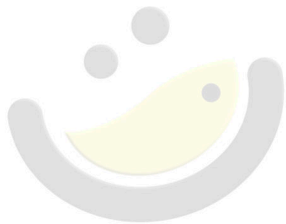
$$= (x^2 - 3)(x^2 - 2x + 5)$$

$$= x^4 - 3x^2 - 2x^3 + 6x + 5x^2 - 15$$

$$= x^4 - 2x^3 + 2x^2 + 6x - 15$$

$$x^4 - 2x^3 + 2x^2 + 6x - 15 = 0 \text{ is the}$$

required eq.



OMG! MATHS!
The poetry of logical ideas.