THEORY OF EQUATIONS

Construct a polynomial equation over rationals of degree 4 whose roots are $\sqrt{3}$ and $1+2i$.

Sol.

Let the required eq. is $f(x)=0$

The Given roots are $\sqrt{3}$, $1+2i$ — (1)

In an eq. with rational Coeff., irrational roots occur in Conjugate Pairs

$\Rightarrow -\sqrt{3}$ is also root of $f(x)$ — (ii)

In an eq. with real coeff, imaginary root occur in Conjugate pairs.

$\Rightarrow 1-2i$ is also root of $f(x)=0$ — (iii)

from (i), (ii) and (iii)

Roots of $f(x)$ are

$$\sqrt{3}, \quad -\sqrt{3}, \quad 1+2i, \quad 1-2i$$

$\Rightarrow$ $(x-\sqrt{3}), (x+\sqrt{3}), (x-(1+2i)), (x-(1-2i))$ are factors of $f(x)$

$\Rightarrow$ Required eq. is

$$f(x) = (x-\sqrt{3})(x+\sqrt{3})(x-(1+2i))(x-(1-2i))$$

$$= (x^2 - 3)[(x-1) - 2i][(x-1) + 2i]$$

$$= (x^2-3)\left((x-1)^2 - (2i)^2\right)$$

$$= (x^2-3)\left(x^2 + 1 - 2x + 4\right) \quad [\because i^2 = -1]$$

$$= (x^2-3)(x^2 - 2x + 5)$$

$$= x^4 - 3x^2 - 2x^3 + 6x + 5x^2 - 15$$

$$= x^4 - 2x^3 + 2x^2 + 6x - 15$$

$x^4 - 2x^3 + 2x^2 + 6x - 15 = 0$ is the required eq.