THEORY OF EQUATIONS

Exp If -2 is a zero of polynomial

$$f(x) = x^4 - (2a+3)x^2 - 2(a-1)x + 12$$

then show that it is a multiple zero and hence find all the zeros of $f(x)$

Sol: $f(x) = x^4 - (2a+3)x^2 - 2(a-1)x + 12$ (Given)

-2 is zero of $f(x)$

$$\Rightarrow \quad f(-2) = 0$$

$$\Rightarrow (-2)^4 - (2a+3)(-2)^2 - 2(a-1)(-2) + 12 = 0$$

$$\Rightarrow 16 - 4(2a+3) + 4(a-1) + 12 = 0$$

$$\Rightarrow 16 - 8a - \cancel{12} + 4a - 4 + \cancel{12} = 0$$

$$\Rightarrow -4a + 12 = 0$$

$$\Rightarrow a = 12/4 = 3.$$

$$\boxed{a = 3}$$

$\therefore\ f(x) = x^4 - 9x^2 - 4x + 12.$

Now -2 is root of $f(x)$ (given)

$\therefore\ x+2$ is factor of $f(x)$

By synthetic division

$$\begin{array}{r|rrrrr} & 1 & 0 & -9 & -4 & 12 \\ -2 & & -2 & 4 & 10 & -12 \\ \hline & 1 & -2 & -5 & 6 & \boxed{0} \end{array}$$

Reduced eq. is.

$$x^3 - 2x^2 - 5x + 6 = 0 \quad — (1)$$

$$(-2)^3 - 2(-2)^2 - 5(-2) + 6 = 0$$

$$-8 - 8 + 10 + 6 = 0$$

$$0 = 0$$

$\therefore$ -2 is root of (1)

By sythetic division.

$$
\begin{array}{r|rrrr}
-2 & 1 & -2 & -5 & 6 \\
 & & -2 & 8 & -6 \\
\hline
 & 1 & -4 & 3 & \boxed{0}
\end{array}
$$

Reduced eq. is

$$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x-3)-1(x-3)=0$$

$$(x-1)(x-3)=0$$

$$x=1,3.$$

$\therefore$ Roots of $f(x)$ are $-2, -2, 1, 3$.

$\therefore$ Multiplicity of -2 is 2.

$\Rightarrow$ -2 is a multiple zero

Hence Proved.