

THEORY OF EQUATIONS

Expt If -2 is a zero of polynomial

$$f(x) = x^4 - (2a+3)x^2 - 2(a-1)x + 1^2$$

then show that it is a multiple zero

and hence find all the zeros of $f(x)$

Sol:

$$f(x) = x^4 - (2a+3)x^2 - 2(a-1)x + 1^2 \quad (\text{Given})$$

-2 is zero of $f(x)$

$$\Rightarrow f(-2) = 0$$

$$\Rightarrow (-2)^4 - (2a+3)(-2)^2 - 2(a-1)(-2) + 12 = 0$$

$$\Rightarrow 16 - 4(2a+3) + 4(a-1) + 12 = 0$$

$$\Rightarrow 16 - 8a - \cancel{12} + 4a - \cancel{4} + \cancel{12} = 0$$

$$\Rightarrow -4a + 12 = 0$$

$$a = 12/4 = 3.$$

$$a = 3$$

$$\therefore f(x) = x^4 - 9x^2 - 4x + 12.$$

Now -2 is root of $f(x)$ (Given)

$\therefore x+2$ is factor of $f(x)$

By synthetic division

The diagram shows the synthetic division process. A vertical line separates the divisor $x + 2$ from the dividend coefficients. The dividend coefficients are 1, 0, -9, -4, 12. The divisor $x + 2$ is written as -2 to its left. The first coefficient 1 is brought down below the line. The product of -2 and 1 is -2 , which is written under the next coefficient 0. The sum of $0 + (-2)$ is -2 , which is written below the line. This process is repeated for each coefficient: $-2 \times -2 = 4$ (sum with -9 is -5), $4 \times -5 = -20$ (sum with -4 is 6), and $-20 \times 6 = -120$ (sum with 12 is 0). The final result is 0 in a box at the bottom right.

-2	1	0	-9	-4	12
		-2	4	10	-12
		-2	-5	6	0

Reduced eq. is.

$$x^3 - 2x^2 - 5x + 6 = 0 \quad \text{--- } ①$$

$$(-2)^3 - 2(-2)^2 - 5(-2) + 6 = 0$$

$$\begin{aligned} -8 - 8 + 10 + 6 &= 0 \\ 0 &= 0 \end{aligned}$$

$\therefore -2$ is root of ①

By synthetic division.

$$\begin{array}{r|rrrrr} -2 & 1 & -2 & -5 & 6 \\ & & -2 & 8 & -6 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

Reduced eq. is

$$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x-3) - 1(x-3) = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1, 3.$$

\therefore Roots of $f(x)$ are $-2, -2, 1, 3$.

\therefore Multiplicity of -2 is 2 .

$\Rightarrow -2$ is a multiple zero

Hence Proved.
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