THEORY OF EQUATIONS

Solve the eq.

$$6x^4 - 13x^3 - 35x^2 - x + 3 = 0$$

which has a root $2-\sqrt{3}$.

Sol. Given eq. is

$$6x^4 - 13x^3 - 35x^2 - x + 3 = 0 \quad \text{--- (1)}$$

Now Given Root is $2-\sqrt{3}$.

which is a irrational root

In an eq. with rational coeff irrational roots occur in conjugate pairs.

$\Rightarrow$ $2+\sqrt{3}$ is also root of ①

$\Rightarrow$ $(x-(2-\sqrt{3})), (x-(2+\sqrt{3}))$ are factors of ①

Now $(x-(2-\sqrt{3}))(x-(2+\sqrt{3}))$ is

$$\left[(x-2)+\sqrt{3}\right]\left[(x-2)-\sqrt{3}\right]$$

$$= (x-2)^2 - (\sqrt{3})^2$$

$$= x^2 + 4 - 4x - 3$$

$$= x^2 - 4x + 1$$

$\therefore$ $x^2 - 4x + 1$ divides (i)

$$
\begin{array}{r|l}
x^2-4x+1 & 6x^4-13x^3-35x^2-x+3 \\
 & \underline{-\cancel{6x^4} \underset{+}{-} 24x^3 \underset{-}{+} 6x^2} \\
 & \quad 11x^3-41x^2-x+3 \\
 & \quad \underline{-\cancel{11x^3} \underset{+}{-} 44x^2 \underset{-}{+} 11x} \\
 & \qquad 3x^2-12x+3 \\
 & \qquad \underline{-\cancel{3x^2} \underset{+}{-} \cancel{12x} \underset{-}{+} \cancel{3}} \\
 & \qquad\qquad \underline{\times}
\end{array}
\qquad
\begin{array}{l}
6x^2+11x \\
\quad +3
\end{array}
$$

$\therefore$ The other roots of eq ① is given by

$$6x^2 + 11x + 3 = 0$$

$$6x^2 + 9x + 2x + 3 = 0$$

$$3x(2x + 3) + 1(2x + 3) = 0$$

$$(3x + 1)(2x + 3) = 0$$

$$x = -1/3,\ -3/2$$