# THEORY OF EQUATIONS

Construct a monic Cubic polynomial $f(x)$ with integral Coeff. s.t. $f(4) = 6$ and $\sqrt{3}+1$ is a root of $f(x) = 0$

Sol. Given eq. is $f(x) = 0$ — (1)

$\sqrt{3}+1$ is root of $f(x)$ (Given)

We know that

In an eq with rational coeff. irrational roots occur in Conjugate pair.

$\Rightarrow -\sqrt{3}+1$ is also root of $f(x)$

Let $\alpha$ is also root of $f(x)$

$\therefore$ Roots are $\alpha, \sqrt{3}+1, -\sqrt{3}+1$

$$\Rightarrow [x - \alpha], [x - (\sqrt{3} + 1)], [x - (-\sqrt{3} + 1)]$$

are factors of $f(x)$

$$\therefore f(x) = (x - \alpha)(x - \sqrt{3} - 1)(x + \sqrt{3} - 1)$$

Put $\underline{\underline{x = 4}}$.

$$f(4) = (4 - \alpha)(4 - \sqrt{3} - 1)(4 + \sqrt{3} - 1)$$

$$6 = (4 - \alpha)(3 - \sqrt{3})(3 + \sqrt{3})$$

$$6 = (4 - \alpha)(9 - 3)$$

$$6 = (4 - \alpha)(6)$$

$$4 - \alpha = 1$$

$$-\alpha = 1 - 4$$

$$\alpha = 3.$$

Now $f(x) = (x-3)\left[\left[(x-1) - \sqrt{3}\right]\left[(x-1) + \sqrt{3}\right]\right]$

$$= (x-3)\left[(x-1)^2 - (\sqrt{3})^2\right]$$

$$= (x-3)\left(x^2+1-2x-3\right]$$

$$= (x-3)(x^2-2x-2)$$

$$= x^3 - 2x^2 - 2x - 3x^2 + 6x + 6$$

$$= x^3 - 5x^2 + 4x + 6$$ is the required eq.