

Theory Of Equations

Polynomials

Horner's Method Of Synthetic Division

find g.c.d. of $f(x) = x^4 + 3x^3 + 2x^2 + x + 1$

and $g(x) = x^3 + x^2 + x + 1$

and write it as $a(x)f(x) + b(x)g(x)$

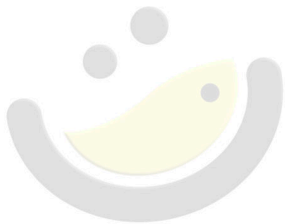
Sol.

$$x^3 + x^2 + x + 1 \quad \sqrt{x^4 + 3x^3 + 2x^2 + x + 1} \quad (x+2)$$
$$\underline{-x^4 + x^3 + x^2 + x}$$

$$2x^3 + x^2 + 1$$
$$\underline{-2x^3 + 2x^2 + 2x + 2}$$

$$-x^2 - 2x - 1 \quad \sqrt{x^3 + x^2 + x + 1} \quad (-x+1)$$
$$\underline{-x^3 + 2x^2 + x}$$

$$-x^2 + 1$$
$$\underline{+x^2 - 2x - 1} \quad -x/2$$
$$2x + 2 \quad \sqrt{-x^2 - 2x - 1}$$
$$\underline{+x^2 + x}$$
$$-x - 1$$



OMG MATHS
The poetry of logical ideas.

$$\text{g.c.d.} = \underline{\underline{x+1}}$$

$$2x+2$$

$$2(x+1)$$

$$\frac{-x-1}{0}$$

In first stage =

$$f(x) = (x+2)g(x) + (-x^2 - 2x - 1) \quad \text{--- (i)}$$

$$g(x) = (-x+1)(-x^2 - 2x - 1) + (2x+2) \quad \text{--- (ii)}$$

$$-x^2 - 2x - 1 = (2x+2)(-x/2 - 1/2) \quad \text{--- (iii)}$$

from (i)

$$\begin{aligned}2x+2 &= g(x) - (-x+1)(-x^2-2x-1) \\ &= g(x) - (-x+1)[f(x) - (x+2)g(x)] \quad [\text{from (i)}]\end{aligned}$$

$$2x+2 = g(x) - (-x+1)f(x) + (-x+1)(x+2)g(x)$$

$$2x+2 = -(-x+1)f(x) + [1 + (-x+1)(x+2)]g(x)$$

$$2x+2 = (x-1)f(x) + [1 + (-x^2-2x+x+2)]g(x)$$

$$2x+2 = (x-1)f(x) + [-x^2-x+3]g(x)$$

$$2(x+1) = (x-1)f(x) + (-x^2-x+3)g(x)$$

$$(x+1) = \frac{x-1}{2} f(x) + \frac{-x^2-x+3}{2} g(x)$$

which is in the form of

where

$$a(x) = \frac{x-1}{2} \quad b(x) = \frac{-x^2-x+3}{2}$$