Theory Of Equations

Polynomials

Horner's Method Of Synthetic Division

find g.c.d. of $f(x) = x^4 + 3x^3 + 2x^2 + x + 1$

and $g(x) = x^3 + x^2 + x + 1$

And write it as $a(x)f(x) + b(x)g(x)$

Sol.

$x^3+x^2+x+1 \,\big)\, \cancel{x^4}+3x^3+2x^2+\cancel{x}+1 \,\big(\, x+2$

$\underline{-\cancel{x^4} - x^3 - x^2 \mp \cancel{x}}$

$\quad \cancel{2x^3}+x^2+1$

$\underline{-\cancel{2x^3} - 2x^2 \mp 2x - 2}$

$-x^2-2x-1 \,\big)\, \cancel{x^3}+x^2+\cancel{x}+1 \,\big(\, -x+1$

$\underline{-\cancel{x^3} - 2x^2 \mp x}$

$\quad -\cancel{x^2}+1$

$\underline{\pm\cancel{x^2} \pm 2x \pm 1}$

$2x+2 \,\big)\, -\cancel{x^2}-2x-1 \,\big(\, -x/2-1/2$

$\underline{\pm \cancel{x^2} \pm x}$

$\quad -x-1$

$+ \quad - \quad x \quad -1$

0

$$g.c.d. = x+1$$

$2x+2$

$2(x+1)$

In first stage =

$$f(x) = (x+2)\,g(x) + (-x^2-2x-1) \quad — \text{(i)}$$

$$g(x) = (-x+1)(-x^2-2x-1) + (2x+2) \quad — \text{(ii)}$$

$$-x^2-2x-1 = (2x+2)\left(-\frac{x}{2}-\frac{1}{2}\right) \quad — \text{(iii)}$$

from (ii)

$$2x+2 = g(x) - (-x+1)(-x^2-2x-1)$$

$$= g(x) - (-x+1)\{f(x) - (x+2)g(x)\} \quad [\text{from (i)}$$

$$2x+2 = g(x) - (-x+1)f(x) + (-x+1)(x+2)g(x)$$

$$2x+2 = -(-x+1)f(x) + [1+(-x+1)(x+2)]g(x)$$

$$2x+2 = (x-1)f(x) + [1+(-x^2-2x+x+2)]g(x)$$

$$2x+2 = (x-1)\,f(x) + [-x^2 - x + 3]\,g(x)$$

$$2(x+1) = (x-1)\,f(x) + (-x^2 - x + 3)\,g(x)$$

$$(x+1) = \frac{x-1}{2}\,f(x) + \frac{-x^2 - x + 3}{2}\,g(x)$$

which is in the form of

$$a(x)\,f(x) + b(x)\,g(x)$$

where $a(x) = \dfrac{x-1}{2}$ $\quad b(x) = -\dfrac{x^2 - x + 3}{2}.$