THEORY OF EQUATIONS

If m and n are integers ≥ 0 and

$f(x)$ and $g(x)$ are polynomials

s.t $(x-\alpha)^m f(x) = (x-\alpha)^n g(x)$, $f(\alpha) \neq 0$

$g(\alpha) \neq 0$

Prove that $m = n$ and $f(x) = g(x)$

Sol. Given:- $(x-\alpha)^m f(x) = (x-\alpha)^n g(x)$ — ①

$f(\alpha) \neq 0 \quad g(\alpha) \neq 0 \quad m \geq 0$

$n \geq 0$

To prove :- $m = n$ and $f(x) = g(x)$

Proof :- $(x-\alpha)^m f(x) = (x-\alpha)^n g(x)$

now Let $m \neq n$

$\Rightarrow m > n$ or $m < n$

Let $m < n$ $\Rightarrow n - m > 0$

Now $(x-\alpha)^m f(x) = (x-\alpha)^n g(x)$ (Given)

$$f(x) = \frac{(x-\alpha)^n}{(x-\alpha)^m} g(x)$$

$$f(x) = (x-\alpha)^{n-m} g(x)$$

$\therefore$ multiplicity of α is $\underline{\underline{n-m}}$.

$\Rightarrow$ α is root of $f(x)$

$\Rightarrow$ $f(\alpha) = 0$ But $f(\alpha) \neq 0$ (given)

So our supposition is wrong.

Hence $m \not< n$

Illy we can prove that $m \not> n$.

Hence $m = n$.

Put $m = n$ in (1)

$$\cancel{(x-\alpha)^m} f(x) = \cancel{(x-\alpha)^m} g(x)$$

$$f(x) = g(x)$$

Hence Proved.