THEORY OF EQUATIONS

Given that $1+i$ is a root of

$x^4 - 2x^3 + 4x - 4 = 0$ solve the eq.

Sol: Given eq. is

$$x^4 - 2x^3 + 4x - 4 = 0 \longrightarrow (1)$$

Also $1+i$ is root of (1)

In an eq. with real Coeff, imaginary roots occur in Conjugate Pairs

$\Rightarrow 1-i$ is also root of ①

$\Rightarrow (x-(1+i)), (x-(1-i))$ are factors of ①

$(x-(1+i))(x-(1-i))$

$= \{(x-1)-i] [(x-1)+i]$

$$= (x-1)^2 - i^2$$

$$= x^2 + 1 - 2x + 1$$

$$= x^2 + 2 - 2x \quad \text{— (ii)}$$

Now (ii) divides (i).

$$
\begin{array}{r|l}
 & x^2 - 2 \\
\hline
x^2 - 2x + 2 & \cancel{x^4} - \cancel{2x^3} + 4x - 4 \\
 & \cancel{x^4} - \cancel{2x^3} + 2x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^2 + 4x - 4 \\
 & -2x^2 + 4x - 4 \\
 & + \quad - \quad + \\
\hline
 & \times
\end{array}
$$

$\therefore$ Other roots of eq. ① are given by

$$x^2 - 2 = 0$$

$$x = \pm\sqrt{2}$$

Roots of eq. ① are

$$1+i,\ 1-i,\ \pm\sqrt{2}$$