

Calculus

Successive Differentiation : Important Questions

$$y = \log(1 + \cos x)$$

Prove that $y_1 y_2 + y_3 = 0$

$$y = \log(1 + \cos x)$$

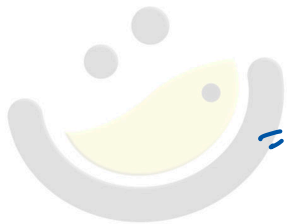
$$y_1 = \frac{1}{1 + \cos x} (-\sin x) = \frac{-\sin x}{1 + \cos x}$$

①

$$y_2 = \frac{(1 + \cos x)(-\cos x) - (-\sin x)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{-\cos x - \cos^2 x - \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{-\cos x - (\sin^2 x + \cos^2 x)}{(1 + \cos x)^2}$$



OMG! MATHS!
The poetry of logical ideas.

$$= \frac{-(\cos x + 1)}{(1 + \cos x)^2} = \frac{-1}{(1 + \cos x)}$$

$$y_2 = \frac{-1}{1 + \cos x} \quad \text{--- (2)}$$

$$y_3 = \frac{(1 + \cos x)(0) - (-1)(-\sin x)}{(1 + \cos x)^2}$$

$$= \frac{-\sin x}{(1 + \cos x)^2} \quad \text{--- (3)}$$

L.H.S
=

$$y_1 y_2 + y_3 = \frac{-\sin x}{(1+\cos x)} \left(\frac{-1}{1+\cos x} \right) +$$

$$\left(\frac{-\sin x}{(1+\cos x)^2} \right)$$

$$= \frac{\sin x}{(1+\cos x)^2} - \frac{\sin x}{(1+\cos x)^2} = 0$$

= R.H.S

Hence $y_1 y_2 + y_3 = 0.$

②. Let $f(x) = \begin{cases} x^2 \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$

show that $f'(x)$ is not continuous at $x = 0$ and $f''(x)$ does not exist at $x = 0$

Sol.

$$f(x) = x^2 \sin 1/x$$

$$f'(x) = x^2 \cos \frac{1}{x} \left(-\frac{1}{x^2}\right) + \sin 1/x (2x)$$

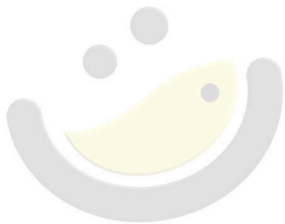
$$= -\cos \sqrt{x} + 2x \sin \sqrt{x}$$

when $x=0$.

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 \sin \sqrt{h} - 0}{h}$$

$$= \lim_{h \rightarrow 0} h \sin \sqrt{h} = 0.$$



OMG! MATHS }
The poetry of logical ideas.

$$f'(x) = \begin{cases} 2x \sin \frac{1}{x} - \cos \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Now.

$$\lim_{x \rightarrow 0} f'(x) = \lim_{x \rightarrow 0} 2x \sin \frac{1}{x} - \cos \frac{1}{x}$$

$$= \lim_{x \rightarrow 0} 2x \sin \frac{1}{x} - \lim_{x \rightarrow 0} \cos \frac{1}{x}$$

$$= 0 - \lim_{x \rightarrow 0} \cos \frac{1}{x} \text{ does not exist}$$

$\Rightarrow f'(x)$ is not continuous at $x=0$.

$\Rightarrow f'(x)$ is not derivable at $x=0$.

$\Rightarrow f''(x)$ does not exist at $x=0$.

Hence Proved.

③

If

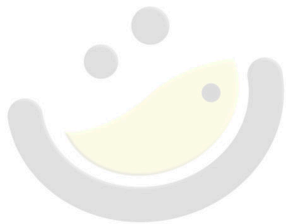
$$x = a(\cot t + t \sin t)$$

$$y = a(\sin t - t \cos t)$$

Compute $\frac{d^2y}{dx^2}$ at $t = \pi/4$

④ If $\sqrt{x} + \sqrt{y} = \sqrt{a}$

find the value of $\frac{d^2y}{dx^2}$ at $x=a$.



OMG { MATHS }

The poetry of logical ideas.