Calculus

Successive Differentiation : Important Questions

$y = \log(1+\cos x)$

Prove that $y_1 y_2 + y_3 = 0$

$y = \log(1+\cos x)$

$$y_1 = \frac{1}{1+\cos x}(-\sin x) = \frac{-\sin x}{1+\cos x} \quad \text{---(1)}$$

$$y_2 = \frac{(1+\cos x)(-\cos x) - (-\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{-\cos x - \cos^2 x - \sin^2 x}{(1+\cos x)^2}$$

$$= \frac{-\cos x - (\sin^2 x + \cos^2 x)}{(1+\cos x)^2}$$

$$= \frac{-(\cos x + 1)}{(1+\cos x)^2} = \frac{-1}{(1+\cos x)}$$

$$y_2 = \frac{-1}{1+\cos x} \quad \text{— (2)}$$

$$y_3 = \frac{(1+\cos x)(0) - (-1)(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{-\sin x}{(1+\cos x)^2} \quad \text{— (3)}$$

L.H.S $= y_1 y_2 + y_3 = \frac{-\sin x}{(1+\cos x)}\left(\frac{-1}{1+\cos x}\right) + \left(\frac{-\sin x}{(1+\cos x)^2}\right)$

$= \frac{\sin x}{(1+\cos x)^2} - \frac{\sin x}{(1+\cos x)^2} = 0$

$=$ R.H.S

Hence $y_1 y_2 + y_3 = 0$.

(2). Let $f(x) = \begin{cases} x^2 \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$

show that $f'(x)$ is not Continous at $x = 0$ and $f''(x)$ does not exist at $x = 0$

Sol. $x \neq 0$.

$f(x) = x^2 \sin 1/x$

$f'(x) = x^2 \cos\frac{1}{x}\left(-\frac{1}{x^2}\right) + \sin 1/x\,(2x)$

$$= -\cos 1/x + 2x \sin 1/x.$$

when $x = 0$.

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}.$$

$$= \lim_{h \to 0} \frac{h^2 \sin 1/h - 0}{h}.$$

$$= \lim_{h \to 0} h \sin 1/h = 0.$$

$$f'(x) = \begin{cases} 2x \sin 1/x - \cos 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Now.

$$\lim_{x \to 0} f'(x) = \lim_{x \to 0} 2x \sin 1/x - \cos 1/x$$

$$= \lim_{x \to 0} 2x \sin 1/x - \lim_{x \to 0} \cos 1/x$$

$$= 0 - \lim_{x \to 0} \cos 1/x \quad \text{does not exist}$$

$\Rightarrow$ $f'(x)$ is not Continous at $x = 0$.

$\Rightarrow$ $f'(x)$ is not derivable at $x = 0$.

$\Rightarrow$ $f''(x)$ does not exist at $x = 0$.

Hence Proved.

③ If $x = a(\cos t + t \sin t)$

$y = a(\sin t - t \cos t)$

Compute $\frac{d^2y}{dx^2}$ at $t = \pi/4$

(4) If $\sqrt{x} + \sqrt{y} = \sqrt{a}$

find the value of $\frac{d^2y}{dx^2}$ at $x = a$.