

EIGEN VALUES AND CAYLEY-HAMILTON THEOREM

Most Important Theorem

State and Prove Cayley-Hamilton Theorem

Statement : Every Square Matrix satisfies its

Characteristics Equation

Proof

Let A is $n \times n$ square Matrix.

Now $|A - \lambda I| = 0$ is ch. eq. of A

$$|A - \lambda I| = a_0 + a_1 \lambda + a_2 \lambda^2 + \dots + a_n \lambda^n = 0 \quad \text{--- (1)}$$

$$\text{T.P.} \quad a_0 + a_1 A + a_2 A^2 + \dots + a_n A^n = 0$$

$$A \cdot \text{adj} A = |A| I$$

$$(A - \lambda I) \text{Adj} (A - \lambda I) = |A - \lambda I| I$$

$$\text{Let } \text{Adj} (A - \lambda I) = b_0 + b_1 \lambda + b_2 \lambda^2 + \dots + b_{n-1} \lambda^{n-1}$$

$$(A - \lambda I) (b_0 + b_1 \lambda + b_2 \lambda^2 + \dots + b_{n-1} \lambda^{n-1}) \quad \left[| \text{adj} A | = |A|^{n-1} \right]$$

$$= \{ a_0 + a_1 \lambda + a_2 \lambda^2 + \dots + a_n \lambda^n \} I \quad \text{(from ①)}$$

Compare the like Coeff of powers of λ

$$A b_0 = a_0 I$$

$$A b_1 - b_0 = a_1 I$$

$$A b_2 - b_1 = a_2 I$$

$$\vdots$$
$$A b_{n-1} - b_{n-2} = a_{n-1} I$$

$$- b_{n-1} = a_n I$$

Now Multiply above eq. with I, A, A^2, \dots, A^n respectively.

$$A b_0 = a_0 I$$

$$A^2 b_1 - A b_0 = A a_1 I$$

$$A^3 b_2 - A^2 b_1 = A^2 a_2 I$$

⋮

$$A^n b_{n-1} - A^{n-1} b_{n-2} = A^{n-1} a_{n-1} I$$

$$- A^n b_{n-1} = A^n a_n I$$

Now add above eqs.

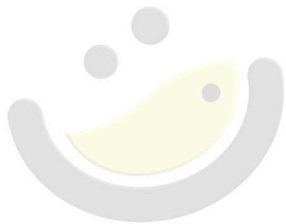


$$0 = a_0 + A a_1 + A^2 a_2 + A^3 a_3 + \dots$$

$$\dots + A^{n-1} a_{n-1} + A^n a_n$$

which is the required result

hence Proved.



OMG MATHS
The poetry of logical ideas.