Rank Of A Matrix

Important Questions (PYQ)

find the value of x so that matrix $\begin{bmatrix} x+p & q & r \\ p & x+q & r \\ p & q & x+r \end{bmatrix}$ is of rank 3.

Sol.

$$\begin{bmatrix} x+p & q & r \\ p & x+q & r \\ p & q & x+r \end{bmatrix}$$

$$C_1 \to C_1 + C_2 + C_3.$$

$$\sim \begin{bmatrix} x+p+q+r & q & r \\ x+p+q+r & x+q & r \\ x+p+q+r & q & x+r \end{bmatrix}$$

$$= (x+p+q+r)\begin{bmatrix} 1 & q & r \\ 1 & x+q & r \\ 1 & q & x+r \end{bmatrix}$$

OMG {MATHS}
The poetry of logical ideas.

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$= (x + p + q + r) \begin{bmatrix} 1 & q & r \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$$

$$= (x + p + q + r)(x^2) \neq 0$$

[$\because$ Rank of Matrix is 3. $|A| \neq 0$].

$$x^2 (x + p + q + r) \neq 0$$

$$x \neq 0, -(p + q + r)$$

OMG { MATHS }
The poetry of logical ideas.

Using elementry transformations, find the Rank of matrix.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

Sol. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

OMG { MATHS }
The poetry of logical ideas.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & -1 & -3 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

$R_3 \to 2R_3 - R_2$

$0 - 0$

$-2 - (-2)$

$-6 - (0)$

$\rho(A) = 3.$

OMG { MATHS } The poetry of logical ideas.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 4 & 6 & 5 \\ 3 & 5 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -6 & -3 \\ 0 & -4 & -2 \end{bmatrix} \qquad \begin{array}{l} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \qquad \begin{array}{l} R_2 \to -\tfrac{1}{3} R_2 \\ R_3 \to -\tfrac{1}{2} R_3 \end{array}$$

OMG{ MATHS }
The poetry of logical ideas.

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

$$\rho(A) = 2.$$

[$\because$ Minor $\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$ of order 2 does not vanish]

OMG { MATHS } The poetry of logical ideas.

⊛ Reduce the Matrix $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ to the normal form and hence, find its rank.

Sol. $A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$

OMG { MATHS } The poetry of logical ideas

$$= \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 1 & 3 & 7 & 5 \\ 1 & 5 & 11 & 6 \end{bmatrix} \qquad C_1 \to \tfrac{1}{2} C_1$$

$$= \begin{bmatrix} 1 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 6 & 8 & 2 \end{bmatrix} \qquad \begin{aligned} R_3 &\to R_3 - R_1 \\ R_4 &\to R_4 - R_1 \end{aligned}$$

OMG {MATHS}
The poetry of logical ideas.

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 4 & 1 \\ 0 & 4 & 4 & 1 \\ 0 & 6 & 8 & 2 \end{bmatrix}
$$

$C_2 \to C_2 + C_1$

$C_3 \to C_3 - 3C_1$

$C_4 \to C_4 - 4C_1$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 4 \\ 0 & 2 & 8 & 6 \end{bmatrix}
$$

$C_2 \leftrightarrow C_4$

OMG {MATHS} The poetry of logical ideas

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_4 \to R_4 - 2R_2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad R_3 \to R_3 - R_2$$

OMG {MATHS}
The poetry of logical ideas.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \leftrightarrow C_4$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

$C_3 \to C_3 - 3C_2$

$C_4 \to C_4 - 4C_2$

OMG{MATHS} The poetry of logical ideas.

$$= \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$\rho(A) = 3.$ Ans

Imp Imp Imp

OMG { MATHS }
The poetry of logical ideas.

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$ then find the matrices P and Q s.t. PAQ is in normal form. Hence find the Rank of Matrix A.

Sol:

$$A = IAI$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OMG { MATHS } The poetry of logical ideas.

$R_2 \to R_2 - 3R_1 \quad R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1 \quad C_3 \to C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OMG { MATHS }
The poetry of logical ideas.

$$R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to -1/2\, R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & -1/2 & 0 \\ 2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \to C_3 - C_2$$

OMG{MATHS} The poetry of logical ideas.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3/2 & -1/2 & 0 \\ 2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix} = PAQ$$

$$\underline{\underline{\rho(A) = 2}}$$

OMG { MATHS }
The poetry of logical ideas.