

Plane Geometry

Parabola

Important Questions (PYQ)

If the normal at a point P of parabola $y^2 = 8x$ meets its axis at G , show that the locus of the middle point of PG is parabola. Find the Co-ordinates of its vertex.

Sol. $y^2 = 8x$ (Given)

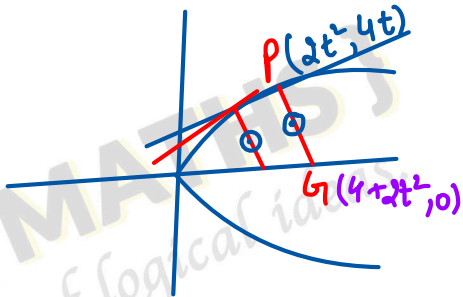
Let P is
 $(at^2, 2at)$

So P is $(2t^2, 4t)$

Normal at P is

$$y = -tx + 2at + at^3$$

$$y = -tx + 4t + 2t^3 \quad \text{--- ①}$$



$$y^2 = 8x$$

Compare with

$$y^2 = 4ax$$

$$4a = 8$$

$$\underline{\underline{a = 2}}$$

Normal meets the axis of parabola at G

i.e. $y=0$

from (1) $0 = -tx + 4t + 2t^3$

$$tx = 4t + 2t^3$$

$$x = 4 + 2t^2$$

$\therefore G$ is $(4 + 2t^2, 0)$ P is $(2t^2, 4t)$

\therefore Mid point of PG is

$$(x_1, y_1) = \left(\frac{2t^2 + 4 + 2t^2}{2}, \frac{4t}{2} \right)$$

$$= (2t^2 + 2, 2t)$$

$$x_1 = 2t^2 + 2 \quad \text{--- ②}$$

$$y_1 = 2t \quad \text{--- ③}$$

from ③ $t = \frac{y_1}{2}$

Put $t = \frac{y_1}{2}$ in ②

$$x_1 = \frac{2 y_1^2}{4} + 2$$

$$x_1 = \frac{y_1^2}{2} + 2$$

$$2x_1 = y_1^2 + 4$$

$$y_1^2 = 2x_1 - 4$$

$$y_1^2 = 2(x_1 - 2)$$

Locus of (x_1, y_1) is $y^2 = 2(x-2)$ — (4)

which is eq. of a parabola

$$\text{let } y = Y \quad x-2 = X.$$

from (4) $y^2 = 2x$ — (5)

which is eq. of a right handed

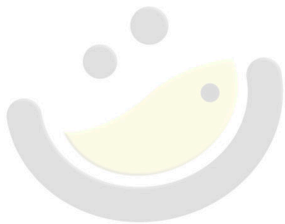
parabola.
Vertex of (5) is (0,0)

$$x = 0 \quad y = 0$$

$$x - 2 = 0 \quad y = 0$$

$$x = 2 \quad y = 0$$

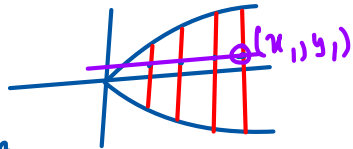
\therefore Vertex is (2,0)



② Prove that the locus of middle points of the system of parallel chords of a parabola is a straight line parallel to axis.

Sol.

Let $y^2 = 4ax$ is the given parabola



Let (x_1, y_1) is the mid-point of any chord of system.

e.g. of chord is

$$yy_1 - 2a(x+x_1) = y_1^2 - 4ax_1$$

$$yy_1 - 2ax - 2ax_1 = y_1^2 - 4ax_1$$

$$yy_1 = y_1^2 - 2ax_1 + 2ax$$

$$y = \frac{2a}{y_1} x + \frac{y_1^2 - 2ax_1}{y_1} \quad \text{--- (1)}$$

Slope of ① is

$$m = \frac{2a}{y_1}$$

$$y_1 = \frac{2a}{m}$$

\therefore Locus of (x_1, y_1) is

$$y = \frac{2a}{m}$$

Which is the eq. of straight line \parallel to x -axis.

Also x -axis is axis of parabola

$$y^2 = 4ax$$

Hence locus of mid point of system of parallel chords of a parabola is a line \parallel to axis.

hence Proved

