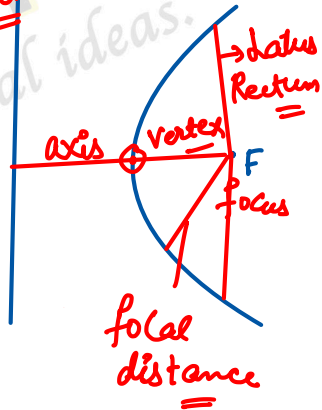# Plane Geometry
Parabola
Full Chapter Revision

Parabola: A Parabola is the locus of a point which moves so that its distance from a fixed point is equal to its distance from a fixed straight line.

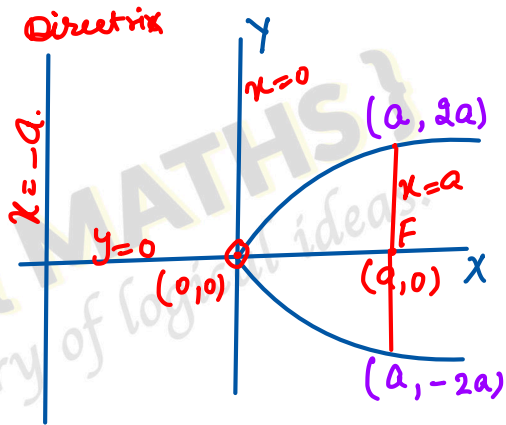$$y^2 = 4ax.$$

Focus $(a, 0)$

Vertex $(0, 0)$

Axis $y = 0$

Directrix $x = -a$

tangent at Vertex $x = 0$

Latus Rectum $x = a.$

Length of length Rectum $= 4a$

end Points of latus rectum $= (a, \pm 2a)$

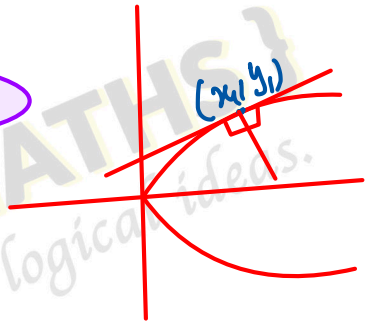| Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex	$(0,0)$	$(0,0)$	$(0,0)$	$(0,0)$
Tangent at vertex	$x = 0$	$x = 0$	$y = 0$	$y = 0$
Focus	$(a,0)$	$(-a,0)$	$(0,a)$	$(0,-a)$
Length of Latus rectum	$4a$	$4a$	$4a$	$4a$
Eq. of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$
end points of L.R.	$(a, \pm 2a)$	$(-a, \pm 2a)$	$(\pm 2a, a)$	$(\pm 2a, -a)$

Tangent to the Parabola

$$yy_1 = 2a(x + x_1)$$

Condition of tangency.

$$y = mx + c \qquad y^2 = 4ax.$$

$$c = \frac{a}{m}$$

Point of Contact

$$\left(\frac{a}{m^2}, \frac{2a}{m} \right)$$

(x_1, y_1)

line $lx + my + n = 0$ $y^2 = 4ax.$

Contion of tangency.

$$am^2 = ln.$$

Point of Contact $\left(\dfrac{n}{l}, \dfrac{-2am}{l} \right)$

Equation of Normal at (x_1, y_1)

$$y - y_1 = \dfrac{-y_1}{2a} (x - x_1)$$

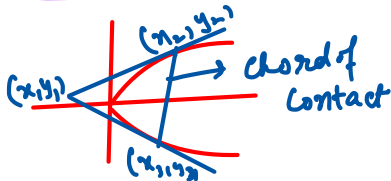# Condition of Normality.

$$y = mx + c \qquad y^2 = 4ax.$$

$$c = -2am - am^3$$

Foot of Normal.

$$(am^2, -2am)$$

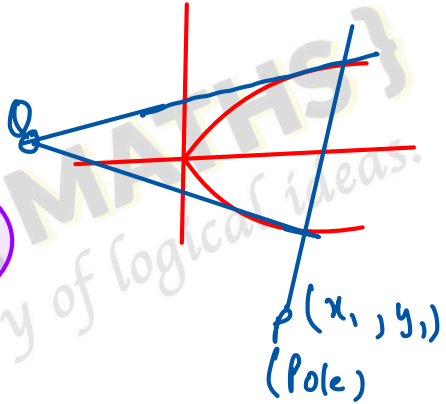## Chord of Contact

$$yy_1 = 2a(x + x_1)$$

Polar :- Path of Q.

$$yy_1 = 2a(x + x_1)$$

Pole

$$\left(\frac{n}{l}, \frac{-2am}{l} \right)$$

Q

$p(x_1, y_1)$
(Pole)

Conjugate lines

$$lx + my + n = 0 \qquad l'x + m'y + n' = 0$$

Conjugate

$$nl' + ln' = 2a\, mm'$$

Joint equation of tangents from exterior Point (x_1, y_1)

$$(y^2 - 4ax)(y_1^2 - 4ax_1) = \left(yy_1 - 2a(x + x_1)\right)^2$$

Equation of chord of parabola having mid-point (x_1, y_1)

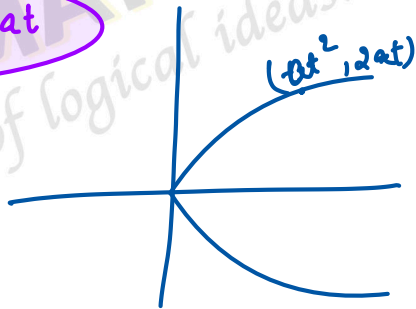$$yy_1 - 2a(x + x_1) = y_1^2 - 4ax_1$$

$$yy_1 - 2ax = y_1^2 - 2ax_1$$

Diameter-

- ‖ to x - axis

- Tangent drawn
  to extrimity of
  diameter is ‖ to chords

- Tangent drawn at the end points
  of ‖ chords meet on diameter

Parametric representation of Parabola.

$$y^2 = 4ax$$

$$x = at^2 \qquad y = 2at$$

equation of Chord
Joint t_1 & t_2

$$(t_1 + t_2)y = 2x + 2at_1t_2$$

$(at^2, 2at)$

Tangent at Point t $(at^2, 2at)$

$y^2 = 4ax.$

$$ty = x + at^2$$

Normal at point 't'

$$y = -tx + 2at + at^3$$

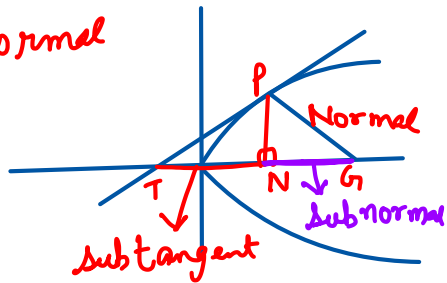The Point of intersection of tangents at t_1 & t_2 $\boxed{(at_1t_2, a(t_1+t_2))}$

The chord Joining t_1 & t_2 is the focal chord of Parabola. if

$$t_1 t_2 = -1$$

Subtangent and Subnormal

Vertex Bisects the Subtangent

subnormal = $\frac{1}{2}$ latus Rectum.